

18020

B. C. A. Examination, May 2018

MATHEMATICS-III

(BCA-406)

(New)

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt questions from all Sections as per instructions.

Section-A**(Very Short Answer Questions)**

Attempt all the five questions. Each question carries 3 marks. Very short answer is required.

$$3 \times 5 = 15$$

1. Express $\frac{2+3i}{4+5i}$ in the form of $x+iy$. 3

2. If $x+iy = \frac{3}{2+\cos\theta+i\sin\theta}$, prove that : 3

$$(x-1)(x-3)+y^2=0.$$

3. Test for convergence $\sum_{n=1}^{\infty} \sin \frac{1}{n}$. 3

4. If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, find the directional derivative of $\frac{1}{r}$ in the direction of \vec{r} . 3

5. For any scalar function $\phi(x, y, z)$, prove that $\text{curl grad } \phi = 0$. 3

Section-B**(Short Answer Questions)**

Attempt any two questions out of the following three questions. Each question carries $7\frac{1}{2}$ marks. Short answer is required. $7\frac{1}{2} \times 2 = 15$

6. Solve $(x+1)\frac{dy}{dx} = x(y^2+1)$. $7\frac{1}{2}$

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7. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, then prove that
 $a + b + c = abc$. 7½
8. Solve $\frac{dy}{dx} = \frac{1+y^2}{\tan^{-1} y - x}$. 7½

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks.

Answer is required in detail. 15×3=45

9. (a) Test the convergence of the series : 7½

$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots$$

- (b) Test for convergence the series whose *n*th term is $\frac{r^n}{n^n}$, $r > 0$. 7½

10. (a) Show that the vector field defined by :

$$\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$$

is irrotational. Find the scalar potential *u* such that $\vec{F} = \text{grad } u = \nabla u$. 7½

- (b) If \vec{E} and \vec{H} are irrotational, prove that $\vec{E} \times \vec{H}$ is solenoidal. <https://www.ccsustudy.com> 7½

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11. (a) Test for the convergence of the series

$$\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$$
 7½

- (b) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$. 7½

12. (a) Solve $y \sec^2 x + (y+7) \tan x \frac{dy}{dx} = 0$. 7½

- (b) Solve $(1+x^2) \frac{dy}{dx} + 2xy = \cos x$. 7½

13. (a) Find the directional derivative of $f(x, y, z) = x^2y^2z^2$ at the point (1, 1, -1) in the direction of the tangent to the curve $x = e^t$, $y = 2 \sin t$, $z = t - \cos t$, at $t = 0$. 7½

- (b) Solve $x(e^y + 4)dx + e^{x+y}dy = 0$. 7½