(20519) Roll No. ....

Total Questions : 13] [Printed Pages : 4

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B.C.A. IInd Semester Examination, May-2019

MATHEMATICS-II

(BCA-201)

(New)

Time : 3 Hrs.] *[M.M. : 75*

Note :– Attempt all the Sections as per instructions.

Section–A

(Very Short Answer Type Questions)

Note :– Attempt all the *five* questions. Each question carries 3 marks.

1. Differentiate finite sets and infinite sets with example.

2. Define trigonometric function, exponential function and logarithmic function.

3. What do you mean by 'Principle of Duality' ?

4. If $u = f\left(\frac{y}{x}\right)$ then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

5. Evaluate the triple integral $\int_0^1 \int_1^2 \int_2^3 dx\, dy\, dz$.

Section–B

(Short Answer Type Questions)

Note :– Attempt any *two* questions out of the following three questions. Each question carries 5 marks.

6. Define equivalence relation. If A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3) (3, 3), (4, 4)}. Then prove that R is an equivalent relation.

7. Find the area of the region bounded by the circle $x^2 + y^2 = a^2$, by double integration.

8. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also find their point of intersection.

Section–C

(Long Answer Type Questions)

Note :– Attempt any *three* questions out of the following five questions. Each question carries 15 marks.

9. (i) If Q be the set of rational numbers and $f : Q \to Q$ be defined by $f(x) = 2x + 3$ then prove that f is bijective. Also find f^{-1}.

(ii) If $f : R \to R$ and $g : R \to R$ be defined by $f(x) = x - 1$ and $g(x) = x^2 + 1$ then find $f \circ g(1)$, $f \circ g(2)$, $g \circ f(2)$, $f \circ f(2)$ and $g \circ g(2)$.

10. (i) Let (L, ≤) is a lattice. If $a, b \in L$ then prove that :

$$a \le b \Leftrightarrow a \wedge b = a$$

and $$a \le b \Leftrightarrow a \vee b = b$$

(ii) Let (L, ≤) be a lattice with least element 0 and greatest element 1. If $a \in L$ then show that :

$$a \vee 1 = 1 \text{ and } a \wedge 1 = a$$

Also $$a \vee 0 = a \text{ and } a \wedge 0 = 0$$

11. (i) Discuss the maxima or minima of the function :

$$u = xy + \left(\frac{a^3}{x}\right) + \left(\frac{a^3}{y}\right)$$

(ii) If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

12. (i) Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar.

(ii) Find the angle of intersection of the spheres :

$$x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$$

and $$x^2 + y^2 + z^2 - 6x - 2y + 2z + 2 = 0$$

13. (i) Evaluate the double integral $\int_0^a \int_0^{\sqrt{(a^2 - x^2)}} x^2 y \, dx \, dy$. Also mention the region of integration involved in this double integral.

(ii) Prove that the value of triple integration :

$$\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} xyz \, dz \, dy \, dx, \text{ is } \frac{1}{48}.$$