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(21216)

Roll No.

BCA-I Sem.

18005

B. C. A. Examination, Dec. 2016

MATHEMATICS-I

(BCA-101)

(New Course)

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt questions from all Sections as per instructions.

Section-A

(Very Short Answer Questions)

Attempt all the five questions of this Section. Each question carries 3 marks. Very short answer is required. 3×5=15

- Show that $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is nilpotent matrix of order 2. 3

- If $y = \log[\log(\log x)]$, find $\frac{dy}{dx}$. 3

- Evaluate $\int \frac{(a^x - b^x)^2}{a^x b^x} dx$. 3

- Calculate the area of parallelogram spanned by the vectors $a = (3, -3, 1)$ and $b = (4, 9, 2)$. 3

- Evaluate $\lim_{n \rightarrow \infty} \frac{\sum n^2}{n^3}$. 3

Section-B

(Short Answer Questions)

Attempt any two questions out of the following three questions. Each question carries 7½ marks. Short answer is required. 7½×2=15

- Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. 7½

- Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$. 7½

8. Expand $\sin x$ in power of x and hence find $\sin 18^\circ$ upto four decimal places. 7½

Section-C

(Detailed Answer Questions)

Attempt any *three* questions out of the following five questions. Each question carries 15 marks. Answer is required in detail. 15×3=45

9. (a) Show that $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist. 5
- (b) Examine the continuity of the function at the indicated point. Also point out the type of discontinuity, if any : 10

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

10. Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and hence find A^{-1} . Also state the Cayley-Hamilton theorem. 15

11. Trace the curve $y^2 = (x-1)(x-2)(x-3)$. 15

12. (a) If $I_n = \int_0^{\pi/4} \tan^n x dx$; prove that $I_{n-1} + I_{n+1} = \frac{1}{n}$.

Deduce the value of I_5 . 10

- (b) Evaluate $\int \frac{xe^x}{(x+1)^2} dx$. 5

13. If $y = \sin(m \sin^{-1} x)$, $|x| < 1$; prove that :

(a) $(1-x^2)y_2 - xy_1 + m^2y = 0$ 5

(b) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$

and find value of $y_n(0)$. 10