

UNIT-5Probability, Sample space, events★ Sample space:-

The set of all possible outcomes of a random experiment is called the sample space or outcome space. It is denoted by 'S'. The elements of sample space are called sample point.

For example:-

- 1- If a coin is tossed, the two outcomes namely head or tail may occur, thus a sample space contains two points head or tail.

$$S = (H, T)$$

- 2- If a dice is tossed the following sample space will be formed -

$$S = (1, 2, 3, 4, 5, 6)$$

- 3- If two coins are tossed the sample space contains -

$$S = (H, H), (H, T), (T, H), (T, T)$$

- 4- If two dice are thrown -

$$(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)$

★ **Event** :- The meaning of event is very clear and to the point when a random experiment is conducted we extract some results this result is called event.

For ex:- If we toss a coin either head will occur or tail will occur, then tossing a coin is an experiment and occurrence of head and tail is an event.

Types of event :-

- 1- Simple and Compound event
- 2- Independent or dependent event
- 3- Complementary event.
- 4- Sure and Impossible event.

★ **Probability** :-

$$\text{Probability} = \frac{\text{No. of favourable cases}}{\text{Total no. of cases.}}$$

Ques:- A bag contain 4 red, 5 green balls. A ball is drawn at random, what is the probability that it is red in colour.

$$\text{Total no. of balls} = 4 + 5 = 9$$

$$\text{No. of red balls} = 4 \text{ balls}$$

$$P = \frac{\text{no. of favourable cases}}{\text{total no. of cases}}$$

$$P = \frac{4}{9}$$

Ques:- There are 38 balls in a bag in which 15 are white, 13 are pink, 10 are black. If a ball is drawn determine the probability of

a) - white ball.

b) - pink ball.

c) - black ball.

d) - black or white ball.

a). Total no. of balls = 38.

No. of white balls = 15

P. of white ball $P(A) = \frac{15}{38}$

b). P. of pink ball $P(B) = \frac{13}{38}$

c). P. of black ball $P(C) = \frac{10}{38} = \frac{5}{19}$

Q1- P. of black or white ball.

$$P(O) = \frac{10+15}{38} = \frac{25}{38}$$

Ques - A bag contains 8 balls of which 5 are red and 3 are black. 2 balls are drawn at random. What is probability that both are black?

Total no. of ways so that 2 balls are drawn = 8C_2

$${}^8C_2 = \frac{8!}{6!2!} = \frac{8 \times 7 \times 6!}{6! \times 2} = 4 \times 7 = 28 \text{ Ans.}$$

Ways of getting 2 black balls = 3C_2
= 3.

P. that 2 balls are black = $\frac{3}{28}$ Ans.

Ques - There are 100 cards. These cards are no. from 1 to 100, one card is drawn at random. What is the probability that the no. on the card is a square.

Total no. of ways = $^{100}C_1 = 100$

Square = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$$P(A) = \frac{10}{100} = \frac{1}{10} \text{ ans.}$$

Ques:- Two dice are thrown and the sum of the no. of the faces up are added. Find the probability that the sum is 10.

total no. of outcomes = 36

$$P(\text{sum is } 10) = \frac{3}{36} = \frac{1}{12} \text{ ans.}$$

Ques- Find the probability of 53 Sunday in a year selected at random.

Total no. of days = 365

total no. of weeks in a year = 52 weeks 1 day

$$P. \text{ of } 53 \text{ Sunday in a year} = \frac{1}{7}$$

Ques:- A bag contains 8 white balls and 4 red balls. 5 balls are drawn at random. What is the probability that two of them are red and 3 are white.

~~The no~~ Total no. of balls = $8 + 4 = 12$.

$$\begin{aligned} \text{Ways of getting white ball} &= {}^8C_3 = \frac{8!}{3!5!} \\ &= \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \times 5!} = 56. \end{aligned}$$

$$\text{ways of getting red balls} = {}^4C_2 = \frac{4!}{2!2!}$$

$$\frac{2 \times 3 \times 2!}{2 \times 1 \times 2!} = 6$$

$$\begin{aligned} \therefore P(\text{getting 3 white and 2 red balls}) &= \frac{{}^8C_3 \times {}^4C_2}{{}^{12}C_5} = \frac{56 \times 6}{12!} \\ &= \frac{56 \times 6}{7!5!} \end{aligned}$$

$$= \frac{56 \times 6}{12 \times 11 \times 10 \times 9 \times 8 \times 7!}$$

$$= \frac{14}{3 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{14}{33}$$

Ques:- 3 horses A, B and C are in a race. A is twice as likely to win as B is twice as likely to win as C. What are their respective probability of winning?

Let the probability of winning $C = u$.
 " " " " $B = 2u$
 " " " " $A = 4u$

ATQ :- Total Probability = 1
 $u + 2u + 4u = 1$

$$7u = 1$$

$$u = \frac{1}{7}$$

$$A = \frac{1}{7}, B = \frac{2}{7}, C = \frac{4}{7} \quad \underline{\text{Ans}}$$

Ques :- A and B throw with 2 dice. If A throw 9. Find the B chances of throwing a highest no. than A.

Total ways when 2 dice are throw = 36
 A's sum = 9.

B's probability more than 9 = 10, 11, 12

$P(B) = (5,5), (6,4), (4,6), (6,5), (5,6), (6,6)$.

$P(B)$ getting more than 9 = $\frac{6}{36} = \frac{1}{6}$ Ans

Ques :- (i) If a pair of dice is thrown. Find the probability. -

- That the sum is neither 8 ~~more~~ nor 10.
- The sum is greater than 12.

a) Sum 8 and 10 = (2, 6), (6, 2), (4, 4), (3, 5),
(5, 3), (5, 5), (6, 4), (4, 6)

P. of sum is 8 and 10 = $\frac{8}{36}$

P (Sum is neither 8 nor 10) = $1 - \frac{8}{36}$

$$= \frac{28}{36} = \frac{7}{9} \text{ Ans}$$

(b) $P = \frac{0}{36} = 0$

(ii) If a pair of dice is thrown. Find the chances of throwing exactly 10 in one throw with 3 dice.

Total ways when 3 dice are thrown = 216

Ques:- 3 coins are tossed simultaneously, what is the probability that they will fall 2 head and 1 tail.

Total no. of ways of 3 coins = $2 \times 2 \times 2 = 8$
 ways of getting 2 head and 1 tail = 3
 (H, H, T), (T, H, H), (H, T, H)

P of getting 2 head and 1 tail = $\frac{3}{8}$ Ans.

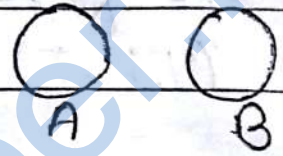
Ques:- Is the statement "The probability of 3 mutually exclusive events A, B and C are $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ respectively" is true?

$$\frac{2}{3} + \frac{1}{4} + \frac{1}{6} = \frac{8+3+2}{12}$$

$$= \frac{13}{12} = 1.09 > 1$$

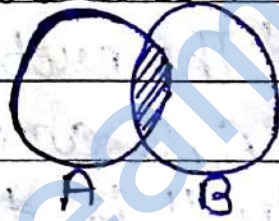
No, because the total probability is more than 1.

★ Addition of theorem of probability :-
 If two events A, B are mutually exclusive
 the probability of occurrence of either
 A or B is the sum of the individual
 probability of A and B.



$$P(A \cup B) = P(A) + P(B)$$

- When events are not mutually exclusive



$$P(A \cup B) = P(A) + P(B) - P(AB)$$

Ques :- A bag contains 30 balls no. from 1 to 30
 1 ball is drawn at random, find the
 probability that the no. of the ball will be
 multiple of 5 or 9.

Sol :- Multiple of 5 = 5, 10, 15, 20, 25, 30.
 $P(\text{multiple of 5}) = \frac{6}{30} = \frac{1}{5}$

Multiple of 9 = 9, 18, 27

$$= \frac{3}{30} = \frac{1}{10}$$

$$P(\text{multiple of 5 or 9}) = P(\text{m of 5}) + P(\text{m of 9})$$

$$= \frac{1}{5} + \frac{1}{10}$$

$$= \frac{2+1}{10} = \frac{3}{10}$$

Ques:- What is the probability of drawing a card of heart or an ace in a single draw from a standard pack of 52 cards?

Sol:- Total cards = 52

$$P(\text{of hearts}) = \frac{13}{52}$$

$$P(\text{of ace}) = \frac{4}{52}$$

$$P(\text{Hearts or ace}) = P(H) + P(\text{Ace}) - P(\text{Heart ace})$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

Ques:- From a pack of 52 cards, a card is drawn at random. What is the probability of getting the card of heart or club or spades.

$$\text{Total cards} = 52$$

$$P \text{ of hearts} = \frac{13}{52}$$

$$P \text{ of club} = \frac{13}{52}$$

$$P \text{ of seven} = \frac{4}{52}$$

$$P(\text{heart or club or seven}) = \frac{13}{52} + \frac{13}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52} = \frac{7}{13}$$

Ques: A bag contains 100 pages no. from 1 to 100. A page is open at random and is selected. Find the probability of the page is multiple of 6 or 10.

Multiple of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96.

$$P(\text{multiple of 6}) = \frac{16}{100}$$

Multiple of 10 = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

$$P(\text{multiple of 10}) = \frac{10}{100}$$

$$P(\text{multiple of 6 or 10}) = \frac{16}{100} + \frac{10}{100} - \frac{3}{100}$$

$$= \frac{23}{100}$$

Ques:- From a pack of 52 cards, 1 card is drawn at random. What is the probability that the card is of spade or an owner.

Total cards = 52.

$$P \text{ of spade} = \frac{13}{52}$$

$$P \text{ of owner} = \frac{16}{52}$$

$$P(\text{spade or owner}) = P(S) + P(O) - P(\text{spade owner})$$

$$= \frac{13}{52} + \frac{16}{52} - \frac{4}{52}$$

$$= \frac{25}{52}$$

Ques:- A dice is thrown. What is the probability of getting a no. less than 5 or an odd no.

Total no. of ways in a dice = 6.

Number less than 5 = 1, 2, 3, 4.

$$P(\text{less than 5}) = \frac{4}{6}$$

odd no. = 1, 3, 5

$$P(\text{odd no.}) = \frac{3}{6}$$

$$P(\text{getting no less than 5 or odd no.}) = P(\text{less than 5}) + P(\text{odd no.}) - P(\text{odd no. less than 5})$$

$$= \frac{4}{6} + \frac{3}{6} - \frac{2}{6} = \frac{5}{6}$$

Ques 8 - Out of no. 1 to 150 cards, 1 no. is selected at random. What is probability that it is divisible by 3 or 5.

No. divisible by 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90, 93, 96, 99, 102, 105, 108, 111, 114, 117, 120, 123, 126, 129, 132, 135, 138, 141, 144, 147, 150.

$$P(\text{divisible by 3}) = \frac{49}{150}$$

No. divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150.

135, 140, 145, 150.

$$P(\text{divisible by } 5) = \frac{30}{150}$$

$$P(\text{divisible by } 3 \text{ or } 5) = \frac{49}{50} + \frac{30}{150} - \frac{10}{150}$$

$$= \frac{69}{150} = \frac{23}{50}$$

Ques:- From 30 tickets mark with the first 30 no. one is drawn at random. It is then replaced and second draw is made. Find the chances that:-

a) In the first draw it is a multiple of 5

b) In the second it is multiple of 3 or 11.

a) Multiple of 5 = 5, 10, 15, 20, 25, 30

$$P(\text{M. of } 5) = \frac{6}{30}$$

Multiple of 7 = 7, 14, 21, 28.

$$P(\text{M. of } 7) = \frac{4}{30}$$

$$P(\text{M. of } 5 \text{ or } 7) = \frac{6}{30} + \frac{4}{30} = \frac{10}{30} = \frac{1}{3}$$

(b) Multiple of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Multiple of 11 = 11, 22.

$$P(\text{m. of } (5 \text{ or } 11)) = \frac{10}{30} + \frac{2}{30} = \frac{12}{30} = \frac{2}{5}$$

★ Multiplication theorem :- If two event A and B are independent the probability that they will both occur is equal to the product of the individual probability symbolically, probability of A and B is equal to probability of multiplication of A and B.

$$P(A \text{ and } B) = P(A) \times P(B).$$

Ques :- If a coin is thrown what is the probability of getting head on third and fourth tosses.

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

$$P(\text{of head on 3rd and 4th toss}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Ques :- 13 cards, each are distributed to 4 players from a pack of 52 cards. What is the probability that particular player get all the 4 ace.

Total cards = 52

ways of getting 13 cards = ${}^{52}C_{13}$

4 card distributed to 4 players = 4C_4

Remaining cards = 9.

= ${}^{40}C_9$

P. of getting all ace = $\frac{{}^4C_4 \times {}^{40}C_9}{{}^{52}C_{13}}$

$$= \frac{1 \times 40!}{39! 9!}$$

52!

39! x 13!

$$= \frac{40! \times 39! \times 13!}{39! \times 9! \times 52!}$$

$$= \frac{40! \times 13 \times 12 \times 11 \times 10 \times 9!}{9! \times 52 \times 51 \times 50 \times 49 \times 48!}$$

$$= \frac{11}{4165} \text{ Ans.}$$

Ques :- A bag contains 10 red and 6 green balls and 4 successive draws are made without replacement. Find the

probability that they are alternative of same colour.

$$P. \text{ of drawing first ball} = \frac{10}{16}$$

$$P. \text{ " " " Second " } = \frac{6}{15}$$

$$P. \text{ " " " third " } = \frac{9}{14}$$

$$P. \text{ " " " fourth " } = \frac{5}{13}$$

$$\begin{aligned} \text{Required probability} &= \frac{10}{16} \times \frac{6}{15} \times \frac{9}{14} \times \frac{5}{13} \\ &= \frac{45}{728} \end{aligned}$$

Ques:- A speak truth in 75% and B in 90% of cases. In what percentage cases are they likely to contradict in each other in stating the same fact.
 'A' speaks truth in 75% cases.

$$P(A) = \frac{75}{100} = \frac{3}{4}$$

$$\begin{aligned} \text{A not speak truth in cases} &= 1 - \frac{3}{4} \\ &= \frac{1}{4} \end{aligned}$$

'B' speaks truth in 20% cases $P(B) = \frac{20}{100} = \frac{1}{5}$

B not speaks truth in cases $P(\bar{B}) = 1 - \frac{1}{5} = \frac{4}{5}$

P. of contradict $P(A) \times P(\bar{B}) = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

P. of contradict $P(\bar{A}) \times P(B) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

Total probability $= \frac{3}{5} + \frac{1}{20}$

$$= \frac{12}{20} = \frac{3}{5}$$

$$= 30\% \text{ } \underline{\underline{\text{Ans}}}$$