

UNIT-4● Combination :-

Selection of the managements of the object is called combination.

For example :-

ABC and BCA are same combination. The total nos. of combination of 'r' object select from n is denoted by :-

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

● Relationship b/w Permutation and Combination :-

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!} = {}^n P_r$$

$${}^n C_n = \frac{1}{1!} = 1 = {}^n P_n$$

$$\boxed{{}^n C_r = \frac{{}^n P_r}{r!}}$$

Ques :- In how many ways can a cricket 11 be chosen out of the batch of 15 players.

- i) There is no restriction on the selection.
- ii) A particular player is always chosen.
- iii) A " " player never be chosen.

$$\begin{aligned}
 \text{f) } {}^{15}C_{11} &= \frac{15!}{11! (15-11)!} = \frac{15!}{11! 4!} \\
 &= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
 &= 105 \times 13 \\
 &= 1365 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } {}^{14}C_{10} &= \frac{14!}{10! 4!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
 &= 77 \times 13 \\
 &= 1001 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } {}^{14}C_{11} &= \frac{14!}{11! (14-11)!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \\
 &= 28 \times 13 \\
 &= 364 \text{ Ans.}
 \end{aligned}$$

Ques:- A committee of 5 is to be form out of 6 gentlemen and 4 ladies in how many ways this each can be done,

- i) Atleast two ladies are included.
- ii) Atleast two ladies are included.

a) Total Gentlemen = 6.

" Ladies = 4

$${}^4C_2 = \frac{4}{\cancel{2} \cancel{2}} = \frac{4 \times 3 \times \cancel{2}}{\cancel{2} \times 2 \times 1} = 2 \times 3 = 6.$$

$${}^6C_3 = \frac{6}{\cancel{3} \cancel{3}} = \frac{6 \times 5 \times \cancel{4} \times 4}{\cancel{3} \times 2 \times 2} = 4 \times 5 = 20$$

$${}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$$

b) ${}^4C_3 \times {}^6C_2$

$$\frac{4}{\cancel{3} \cancel{1}} \times \frac{6}{\cancel{2} \cancel{4}} = \frac{4 \times \cancel{3}}{\cancel{3}} \times \frac{6 \times 5 \times \cancel{4}}{\cancel{4} \times 2 \times 1} = 4 \times 15 = 60.$$

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(iii) ${}^4C_4 \times {}^6C_1$

$$\frac{4}{\cancel{4} \cancel{3} \cancel{2} \cancel{1}} \times \frac{6}{\cancel{5} \cancel{1}} = \frac{4 \times \cancel{1}}{\cancel{4}} \times \frac{6 \times \cancel{1}}{\cancel{1}} = 1 \times 6 = 6.$$

$$\frac{4 \times \cancel{1}}{\cancel{4}} \times \frac{6 \times \cancel{1}}{\cancel{1}} = 1 \times 6 = 6.$$

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$${}^n C_4 = 120 + 60 + 6 = 186 \text{ Ans.}$$

(ii) at most 2 Ladies:-

a) ${}^4C_2 \times {}^6C_3$

$$\frac{4}{\cancel{2} \cancel{2}} \times \frac{6}{\cancel{3} \cancel{3}} = \frac{4 \times 3 \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 1} \times \frac{6 \times 5 \times 4 \times \cancel{3}}{\cancel{3} \times \cancel{3} \times 2 \times 1} = 6 \times 20 = 120.$$

$$= \frac{4 \times 3 \times \cancel{2}}{\cancel{2} \times \cancel{2} \times 1} \times \frac{6 \times 5 \times 4 \times \cancel{3}}{\cancel{3} \times \cancel{3} \times 2 \times 1} = 6 \times 20 = 120.$$

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b) ${}^4C_1 \times {}^6C_4$

$$\frac{4}{\cancel{3} \cancel{2} \cancel{1}} \times \frac{6}{\cancel{2} \cancel{4}} = \frac{4 \times \cancel{3}}{\cancel{3}} \times \frac{6 \times 5 \times \cancel{4}}{\cancel{4} \times 2} = 4 \times 15 = 60.$$

$$= \frac{4 \times \cancel{3}}{\cancel{3}} \times \frac{6 \times 5 \times \cancel{4}}{\cancel{4} \times 2} = 4 \times 15 = 60.$$

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$$c) \quad {}^4C_6 \times {}^6C_5$$

$$\frac{4}{10} \times \frac{6}{4 \times 5}$$

$$\frac{4}{4} \times \frac{6 \times \cancel{5}}{\cancel{5}} = 1 \times 6 = 6$$

$${}^n C_{91} = 120 + 60 + 6 = 186 \quad \underline{\text{Ans}}$$

Quest:- A bag contains 5 black, 3 white and 2 Red balls. In how many ways:-

i) 3 balls be drawn

ii) 3 black balls are drawn.

iii) 2 black and 2 white balls are drawn.

iv) 3 black and 2 white and 1 red balls are drawn.

$$i) \quad {}^{10}C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = \frac{10 \times 9 \times \cancel{8} \times \cancel{17}}{\cancel{17} \times \cancel{8} \times 2 \times 1}$$

$$= 120 \quad \underline{\text{Ans}}$$

$$ii) \quad {}^5C_3 = \frac{5 \times 4 \times 3}{1 \times 2 \times 3} = \frac{5 \times 4 \times \cancel{3}}{\cancel{3} \times 2 \times 1} = 10 \quad \underline{\text{Ans}}$$

$$iii) \quad {}^5C_2 * {}^3C_2$$

$$\frac{5 \times 4}{1 \times 2} \times \frac{3 \times 2}{1 \times 2} = \frac{5 \times 4 \times \cancel{17}}{\cancel{17} \times \cancel{2} \times 1} \times \frac{3 \times \cancel{2}}{\cancel{2}}$$

$$10 \times 3 = 30 \quad \underline{\text{Ans}}$$

$$\text{iv) } {}^5C_3 \times {}^3C_2 \times {}^2C_1$$

$$\frac{15}{1 \times 2 \times 3} \times \frac{3}{1 \times 2} \times \frac{2}{1}$$

$$\frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \frac{3 \times 2}{2} \times \frac{2 \times 1}{1}$$

$$10 \times 3 \times 2 = 60 \text{ Ans.}$$

* Permutation P-

The word permutation means only of the base in which a set of object can be arrange.

$${}^n P_r = \frac{n!}{(n-r)!}$$

Q1- Evaluate :- ${}^{12}P_4$

$${}^{12}P_4 = \frac{12!}{12-4} = \frac{12!}{8} = \frac{12 \times 11 \times 10 \times 9 \times \cancel{8!}}{\cancel{8!}}$$

$$= 11880 \text{ Ans.}$$

$${}^{75}P_2 = \frac{75!}{75-2} = \frac{75 \times 74 \times \cancel{73!}}{\cancel{73!}}$$

$$= 5550 \text{ Ans.}$$

$${}^8P_0 = \frac{8!}{8-0} = \frac{8!}{0} = \frac{8!}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans.}$$

Ques :- The word 'DELHI' contains. How many words with or without meaning can be formed by using the letters 'DELHI'.

$${}^5P_5 = \frac{5!}{5-5} = \frac{5!}{0} = \frac{5!}{1}$$

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 20 \times 6 = 120 \text{ Ans.}$$

Ques :- In how many ways can 8 students be seated?

i) In circle

ii) In a line

$${}^8P_7 = \frac{8!}{8-7} = \frac{8!}{1} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1}$$

$$= 40320 \text{ Ans.}$$

$${}^8P_8 = \frac{8!}{8-8} = \frac{8!}{0} = \frac{8!}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans.}$$

Ques :- i) How many permutations can be made out of the letters 'TRIANGLE'?

$${}^8P_8 = \frac{8!}{8-8} = \frac{8!}{0} = \frac{8!}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans}$$

ii) How many arrangements can be done that start with T and end with E.

$${}^6P_6 = \frac{6!}{6-6} = \frac{6!}{0} = \frac{6!}{1}$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 30 \times 24 = 720 \text{ Ans}$$

Ques:- How many words can be formed of the word 'DAUGHTER' so that

i) The vowels always come together.

ii) The vowels are never together.

iii) AUE D G H T R

$${}^6P_6 = \frac{6!}{6-6} = \frac{6!}{0} = \frac{6!}{1}$$

$$= 5 \times 4 \times 3 \times 2 \times 1$$

$$= 30 \times 24 = 720 \text{ Ans}$$

$$iv) {}^8P_8 = \frac{8!}{8-8} = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans}$$

Ques:- In a lottery each ticket has 5 digit no. 0-9 on it.

a) You win if your ticket has the digit in any order. What are your chances of winning?

b) You could win only if your ticket has the digit in the required order. What are the chances winning.

a) 0-9, total = 10

$$n = 10$$

$$r = 5$$

$${}^{10}C_5 = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$$

$$= 6 \times 42 = 252 \text{ Ans}$$

b) ${}^{10}P_5 = \frac{10!}{\cancel{5!}} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$

$$= 720 \times 42 = 30240 \text{ Ans}$$

Ques:- A museum has 7 paintings by Mr. X and want to arrange 3 of them on the same wall. How many ways this can be done.

$${}^7P_3 = \frac{7!}{\cancel{4!}} = \frac{7 \times 6 \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$$

$$= 42 \times 5 = 210 \text{ Ans}$$

Ques P- A bag contains 10 black ball, 6 yellow ball and 4 blue ball. In how many ways:-

- 7 balls be drawn.
- 5 black balls be drawn.
- 3 black balls 3 yellow ball be drawn.
- 4 black ball 4 yellow ball and 3 blue ball are drawn.

$$a) {}^{20}C_7 = \frac{{}^{20}P_7}{7!} = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 60 \times 19 \times 17 \times 12 = 2,32,560 \text{ Ans}$$

$$b) {}^{10}C_5 = \frac{{}^{10}P_5}{5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 10 \times 14 = 252 \text{ Ans}$$

$$c) {}^{10}C_3 \times {}^6C_3$$

$$\frac{{}^{10}P_3}{3!} \times \frac{{}^6P_3}{3!}$$

$$\frac{10 \times 9 \times 8}{7 \times 6 \times 5} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$$

$$120 \times 20 = 2400 \text{ Ans}$$

$$d) {}^{10}C_4 \times {}^6C_4 \times {}^4C_3$$

$$\frac{{}^{10}P_4}{4!} \times \frac{{}^6P_4}{4!} \times \frac{{}^4P_3}{3!}$$

$$\frac{10 \times 9 \times 8 \times 7 \times 6}{10 \times 9 \times 8 \times 7 \times 6} \times \frac{6 \times 5 \times 4}{2 \times 1 \times 4} \times \frac{4 \times 3}{18}$$

$$30 \times 7 \times 15 \times 4$$

$$210 \times 60 = 12600 \text{ Ans.}$$

Ques: How many 5 letters words can be form the letter 'EQUATIONS'.

$${}^9P_5 = \frac{9!}{9-5} = \frac{9!}{4}$$

$$= 9 \times 8 \times 7 \times 6 \times 5 \times 4$$

$$= 72 \times 42 \times 5$$

$$= 360 \times 42$$

$$= 15120 \text{ Ans.}$$

Ques: How many words with or without can be form using all the letters of the word 'EQUATIONS' using each letters exactly one.

$${}^8P_8 = \frac{8!}{8-8} = \frac{8!}{0} = \frac{8!}{1}$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 40320 \text{ Ans.}$$