

UNIT-2

★ Measurement of Central Tendency!

1- Concept of Central Tendency:-

There are two types of average -

- i) Mathematical average.
- ii) Positional average.

1- Mathematical average:-

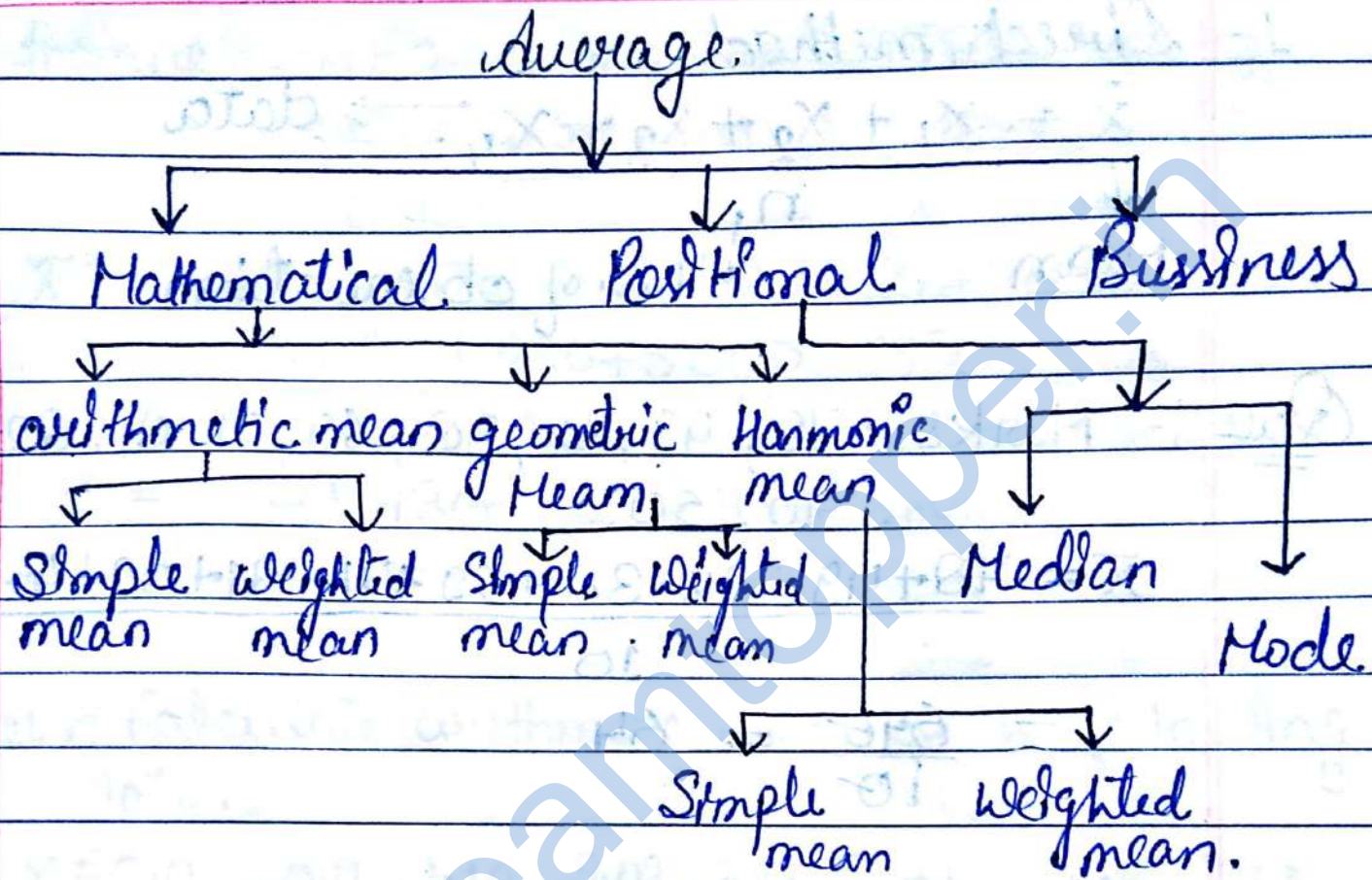
These are calculated values of the series that is why they are classified under this they are -

- a) Arithmetic mean / Average.
- b) Geometric mean.
- c) Harmonic mean.

2- Positional average:-

They are located the position of the average in the series that is why is called positional average.

- a) Median
- b) Mode



* Essential of a good central Tendency :-

- 1- Well define.
- 2- Easy to understand.
- 3- Based on all item of series.
- 4- Good representation of data.
- 5- free from the effect of extreme value.

• Calculation of arithmetic mean in Individual Series :-

1- Direct method.

$$\bar{X} = \frac{X_1 + X_2 + X_3 + X_4 \rightarrow \text{data}}{n}$$

↓
Mean

n ↓
No. of observation

Ques :- Marks :- 78, 49, 82, 38, 69, 71, 81, 82, 40, 50.

$$\bar{X} = \frac{78 + 49 + 82 + 38 + 69 + 71 + 81 + 82 + 40 + 50}{10}$$

$$= \frac{640}{10} = 64$$

Ques :- Calculate the arithmetic mean from the following data by testing the lifetime of bulb.

lifetime :- 967, 889, 940, 922, 952.

$$\bar{X} = \frac{967 + 889 + 940 + 922 + 952}{5}$$

$$= \frac{4,670}{5} = 934$$

Ques :- Calculate the arithmetic mean from the following data.

Income:- 4780, 5760, 4690, 4750, 4840,
4920, 4100, 4850, 4850, 5050,
5950.

$$\bar{X} = \frac{4780 + 5760 + 4690 + 4750 + 4840 + 4920 + 4100 + 4850 + 5050 + 5950}{10}$$

$$\bar{X} = \frac{49,730}{10} = 4,973 \text{ Ans.}$$

Ques:- Calculate arithmetic mean of the following data.

$X = 0.01, 0.11, 1.10, 2.02, 2.23, 2.91, 3.12, 3.19,$
 $3.90, 4.02.$

$$\bar{X} = \frac{0.01 + 0.11 + 1.10 + 2.02 + 2.23 + 2.91 + 3.12 + 3.19 + 3.90 + 4.02}{10}$$

$$= \frac{22.61}{10} = 2.261 \text{ Ans.}$$

2- Shortcut method:-

$$\bar{X} = A + \frac{\sum d_i}{N}$$

N 50.1

21.1

20.6

82.6

20.8

PP.8

8

2

01

Ques:- Calculate arithmetic mean by shortcut method.

S.no.	X.
1	.82
2	.96
3	1.01
4	1.81
5	1.90
6	2.09
7	2.11
8	2.98
9	3.05
10	3.99

S.no.	X.	$du(x-A)$
1	.82	-1.08
2	.96	-1.06
3	1.01	-0.89
4	1.81	-0.09
5	1.90	0
6	2.09	0.19
7	2.11	0.21
8	2.98	1.08
9	3.05	1.15
10	3.99	2.09

$$\Sigma du = 1.72$$

$$\begin{aligned}\bar{x} &= 1.90 + \frac{1672}{1000} \\ &= 1.90 + 0.172 \\ &= 2.072 \text{ Ans.}\end{aligned}$$

Ques:- Calculate the arithmetic mean by shortcut method.

Roll no.	Marks	$dx(x-A)$
1	30	11
2	20	1
3	20	9
4	33	14
5	19	0
6	22	3
7	26	7
8	18	-1
9	20	1
10	14	-5
		$\Sigma dx = 40$

$$\bar{x} = A + \frac{\Sigma dx}{N} = 19 + \frac{40}{10} = 23 \text{ Ans.}$$

~~Ques~~ Calculation of arithmetic mean in discrete series.

i) Direct method :-

$$\bar{X} = \frac{X_1 F_1 + X_2 F_2 + X_3 F_3 + \dots}{\sum F}$$

or

$$\bar{X} = \frac{\sum XF}{\sum F}$$

Ques :- Find the mean from the following data

X	F	X	F	XF
8	3	8	3	24
9	5	9	5	45
10	8	10	8	80
11	12	11	12	132
12	7	12	7	84
13	5	13	5	65
			$\sum F = 40$	$\sum XF = 430$

$$\bar{X} = \frac{430}{40} = \frac{43}{4} = 10.75$$

- Shortcut method in discrete series:-

$$\bar{x} = A + \frac{\sum fdu}{\sum f}$$

Q:-

Height	No. of Student	$du (x-A)$	fdu
60	2	-7	-14
61	3	-6	-18
63	4	-4	-16
64	9	-3	-27
66	10	-1	-10
67	20 = A	0	0
68	17	1	17
69	14	2	28
70	5	3	15
71	3	4	12
72	3	5	15
73	2	6	12
	$\sum f = 100$		$\sum fdu = 6$

$$\bar{x} = A + \frac{\sum fdu}{N} = \cancel{67} + \frac{6}{100}$$

$$= 67 + 0.06 = 67.06$$

Ques:- Find the A.M. from the following data by Shortcut method.

X	f	dn	f _{dn}	
9	2	4	8	
8	3	3	24	
7	4	2	8	
6	6	1	6	
5	11	0	0	$\bar{X} = A + \frac{\sum f_{dn}}{\sum f}$
4	10	-1	-10	$= 5 - \frac{19}{50}$
3	6	-2	-12	$= 5 - 0.38$
2	5	-3	-15	$= 0.12$ <u>Ans</u>
1	2	-4	-8	
0	1	-5	-5	
	50		-19	

Ques:- From the following information calculate mean by shortcut method.

Marks	No. of Student	dn	f _{dn}	
20	8	-20	-160	
30	12	-10	-120	
40	20	0	0	
50	10	10	100	
60	6	20	120	
70	4	30	120	
	60		60	

$$\bar{X} = \frac{A + \sum fcm}{\sum f}$$

$$= \frac{40 + 60}{60} = 40 + 1 = 41 \text{ Ans}$$

Ques:- Calculation of A.M. in Continuous series -

i) Direct Method.

$$\bar{X} = \frac{\sum FX}{\sum f}$$

CF	f	$X = \frac{L+U}{2}$	FX	
0-10	9	5	45	$\bar{X} = \frac{\sum FX}{\sum f}$ $= \frac{4570}{100}$ $= 45.7 \text{ Ans}$
10-20	7	15	105	
20-30	11	25	275	
30-40	15	35	525	
40-50	25	45	1125	
50-60	22	55	1210	
60-70	9	65	585	
70-80	5	75	375	
80-90	5	85	425	
	100		4570	

Ques:- From the following data calculate A.M.

C.I	F	X	FX
0-10	5	5	25
10-20	10	15	150
20-30	25	25	625
30-40	30	35	1050
40-50	20	45	900
50-60	10	55	550
	<u>100</u>		<u>3300</u>

$$\bar{X} = \frac{\sum fdx}{\sum f}$$

$$= \frac{3300}{100}$$

$$= 33 \text{ Ans}$$

Ques:- Calculation of A.M. by shortcut method in continuous series.

$$\bar{X} = A + \frac{\sum fdx}{\sum f}$$

C.I	F	X	dx = (X-A)	fdx
0-10	10	5	-40	-400
10-20	10	15	-30	-300
20-30	14	25	-20	-280
30-40	36	35	-10	-360
40-50	44	45	0	0
50-60	30	55	10	300
60-70	24	65	20	480
70-80	12	75	30	360
80-90	16	85	40	640
90-100	4	95	50	200
	<u>200</u>			<u>640</u>

$$\bar{X} = \frac{45 + 640}{200} = \frac{45}{3.2} = 40.2 \text{ Ans}$$

Ques: Calculate arithmetic mean by shortcut method.

C-I	F	X	du	Fdu	
0-10	4	5	-30	-120	
10-20	8	15	-20	-160	
20-30	11	25	-10	-110	$\bar{X} = A + \frac{\sum Fdu}{\sum F}$
30-40	15	35	0	0	$= 35 - \frac{20}{60}$
40-50	11	45	10	110	$= 35 - 0.33$
50-60	7	55	20	140	$= 34.67 \text{ Ans}$
60-70	4	65	30	120	
	60			-20	

Ques: - Shortcut method

C-I	F	X	du	Fdu	
0-10	5	5	-20	-100	
10-20	10	15	-10	-100	$\bar{X} = 25 + \frac{000}{100}$
20-30	25	25	0	0	$= 25 + 0$
30-40	30	35	10	300	$= 33 \text{ Ans}$
40-50	20	45	20	400	
50-60	10	55	30	300	
	100			800	

Ques:- Calculate d.M. from the following data by shortcut and direct method.

C-5	F	X	dn	Fdn	FX
0-5	15	2.5	-20	-300	37.5
5-10	24	7.5	-15	-360	180
10-15	20	12.5	-10	-200	350
15-20	40	17.5	-5	-200	700
20-25	50	22.5	0	0	1125
25-30	30	27.5	5	150	825
30-35	25	32.5	10	250	812.5
35-40	20	37.5	15	300	750
40-45	10	42.5	20	200	420.5
45-50	8	47.5	25	200	380
	250			-40	5505

(i) By shortcut method,

$$\bar{x} = 22.5 - \frac{40}{250} = 22.5 - 0.16$$

$$\bar{x} = 22.34$$

(ii) By direct method,

$$\bar{x} = \frac{\sum Fdn}{\sum F} = \frac{5505}{250} = 22.34$$

Ques:- Calculate A.M. by direct method and shortcut method.

C-I	F	X	du	Fdu	Fx
45-50	18	47.5	-25	-450	855
50-55	21	52.5	-20	-420	1102.5
55-60	31	57.5	-15	-465	1782.5
60-65	45	62.5	-10	-450	2812.5
65-70	85	67.5	-5	-425	5737.5
70-75	26	72.5	0	0	1885
75-80	22	77.5	5	110	1705
80-85	18	82.5	10	180	1485
85-90	40	87.5	15	600	3500
90-95	5	92.5	20	100	462.5
95-100	5	97.5	25	125	487.5
	266			-845	18440

$$\text{Q.1) } \bar{X} = \frac{18440}{266} = 69.323$$

$$\begin{aligned} \text{(ii) } \bar{X} &= A + \frac{\sum Fdu}{\sum F} = 72.5 - \frac{845}{266} \\ &= 72.5 - 3.17 = 69.33 \text{ Ans} \end{aligned}$$

- Calculation of A.M. in continuous series by step-deviation method.

$$\bar{X} = A + \frac{\sum fd'u}{\sum f} \times i$$

Salary	No. of employees	X	du	d'u	fd'u
1500 - 2500	13	2000	-4000	-4	-52
2500 - 3500	12	3000	-3000	-3	-36
3500 - 4500	20	4000	-2000	-2	-40
4500 - 5500	32	5000	-1000	-1	-32
5500 - 6500	16	6000	0	0	0
6500 - 7500	15	7000	1000	1	15
7500 - 8500	12	8000	2000	2	24
8500 - 9500	8	9000	3000	3	24
9500 - 10,500	4	10,000	4000	4	16
	150				23

$$\bar{X} = 6000 + \frac{23 \times 1000}{150}$$

$$= 6000 + 23 \times 100 = 6000 + 153.333$$

$$= \underline{\underline{6,153.333 \text{ Am.}}}$$

Ques :- Calculate A.M. by step deviation method.

C-I	F	X	du	d'u	Fd'u
0-10	5	5	-20	-2	-10
10-20	10	15	-10	-1	-10
20-30	25	25	0	0	0
30-40	30	35	10	1	30
40-50	20	45	20	2	40
50-60	10	55	30	3	30
	100				80

$$\bar{X} = 25 + \frac{80}{100} \times 10 = 33 \text{ Am}$$

Ques :- Calculation of A.M. In case of unequal interval.

C-I	f	X	Fx
5-10	15	7.5	112.5
10-18	22	14	308
20-40	30	30	900
50-100	24	75	1800
100-250	22	175	3850
250-500	20	375	7500
500-1000	15	750	11250
1000-2000	5	1500	7500
	153		33,220.5

$$\begin{aligned} \bar{X} &= \frac{\sum fx}{\sum f} \\ &= \frac{33220.5}{153} \\ &= 217.129 \text{ Am} \end{aligned}$$

Ques Calculation of A.M. in case of Inclusive series.

Marks	F	C-I	X	FX
10-19	5	9.5-19.5	14.5	72.5
20-29	8	19.5-29.5	24.5	196
30-39	12	29.5-39.5	34.5	414
40-49	8	39.5-49.5	44.5	356
50-59	7	49.5-59.5	54.5	381.5
	40			1420.0

$$\bar{X} = \frac{1420}{40} = \frac{71}{2}$$

$$= 35.5 \text{ Ans.}$$

Ques Calculation of A.M. by step deviation method

C-I	F	X	d'u	d'u	Fd'u
0-10	4	5	-20	-2	-8
10-20	6	15	-10	-1	-6
20-30	10	25	0	0	0
30-40	15	35	10	1	15
40-50	8	45	20	2	16
50-60	7	55	30	3	21
	50				38

$$\bar{X} = 25 + \frac{38}{50} = 32.6 \text{ Ans.}$$

Ques:- C-I	f	C-I	FX	FX
below-10	4	0-10	5	20
10-30	6	10-30	20	120
30-60	10	30-60	40	400
60-100	7	60-100	80	560
above-100	3	100-150	125	375
	30			1475

$$\bar{X} = \frac{\sum fu}{\sum f} = \frac{1475}{30} = 49.16 \text{ Ann.}$$

- Arithmetic mean in case of commulative frequency distribution -

Height	No. of tree (cf)	f	C-I	X	FX
less than 7	25	25	0-7	8.5	212.5
" 14	57	32	7-14	10.5	336
" 21	92	35	14-21	17.5	612.5
" 28	134	42	21-28	24.5	1029
" 35	216	82	28-35	31.5	2583
" 42	287	71	35-42	38.5	2733.5
" 49	341	54	42-49	45.5	2457
" 56	360	19	49-56	52.5	997.5
		360			10,836

dm	Fdm
-21	-525
-14	-440
-7	-245
0	0
7	574
14	994
21	1134
28	532
	6000
	2016

(i) $\bar{X} = \frac{\sum fx}{\sum f} = \frac{10036}{360}$
 $= 30.1$ Ans.

(ii) $\bar{X} = A + \frac{\sum fdx}{\sum f}$

$\bar{X} = 24.5 + \frac{2016}{360}$
 $= 24.5 + 5.6$
 $= 30.1$ Ans.

Ques :- Calculate A.M. from the following data.

C-I.	f	X	FX
10-20	4	15	60
20-40	10	30	300
40-70	26	55	1430
70-120	8	95	760
120-200	2	160	320
	50		2,870

$\bar{X} = \frac{\sum fx}{\sum f}$
 $= \frac{2870}{50}$
 $= 57.4$ Ans.

Ques ^o	No. of Students	f	C-I	X	FX	du	fdu
above-0	80	3	0-10	5	15	-40	-120
" - 10	70 77	5	10-20	15	75	-20	-150
" - 20	72	7	20-30	25	175	-20	-140
" - 30	65	10	30-40	35	350	-10	-100
" - 40	55	12	40-50	45	540	0	0
" - 50	43	15	50-60	55	825	10	150
" - 60	28	12	60-70	65	780	20	240
" - 70	16	6	70-80	75	450	30	180
" - 80	10	2	80-90	85	170	40	80
" - 90	8	8	90-100	95	760	50	400
above-100	0	0	100-110	105	0	60	0
		80			4,140		540

$$\bar{X} = \frac{\sum FX}{\sum f} = \frac{4140}{80} = 51.75$$

$$\bar{X} = A + \frac{\sum fdu}{\sum f} = 45 + \frac{540}{80} = 45 + 6.75$$

$$= 51.75 \text{ Ans}$$

Ques^o:- Calculation of A.M. from the following method, direct method, shortcut method and step deviation.

C-I	F	X	FX	dm	f _{dm}	$d'u = \frac{dm}{5}$	f _{d'u}
45-50	18	47.5	855	-25	-450	-5	-90
50-55	21	52.5	1102.5	-20	-420	-4	-84
55-60	31	57.5	1782.5	-15	-465	-3	-93
60-65	45	62.5	2,812.5	-10	-450	-2	-90
65-70	35	67.5	2,362.5	-5	-175	-1	-35
70-75	26	72.5	1,885	0	0	0	0
75-80	22	77.5	1,705	5	110	1	22
80-85	18	82.5	1,485	10	180	2	36
85-90	14	87.5	1,225	15	210	3	42
90-95	5	92.5	462.5	20	100	4	20
95-100	5	97.5	487.5	25	125	5	25
	240		16,165		1235		-247

i) direct method:- $\bar{x} = \frac{\sum FX}{\sum F} = \frac{16,165}{240} = 67.354$.

ii) Shortcut method:- $\bar{x} = A + \frac{\sum f d_m}{\sum F} = 72.5 - \frac{1235}{240}$

$$\bar{x} = 67.355$$

iii) Step-deviation method:-

$$\bar{x} = A + \frac{\sum d'u}{\sum F} \times C = 72.5 + \left(\frac{-247}{240} \right) \times 5$$

$$= 72.5 - 5.14 = 67.36$$

Ques :- Calculate A.M. from the following data by step deviation method.

Mid values (x)	F	dx	$d'u = \frac{dx}{20}$	fd'u
115	2	-60	-3	-6
135	4	-40	-2	-8
155	6	-20	-1	-6
<u>175</u>	9	0	0	0
195	10	20	1	10
215	9	40	2	18
235	7	60	3	21
255	4	80	4	16
	<u>50</u>			<u>45</u>

$$\bar{X} = A + \frac{\sum fd'u}{\sum F} \times C = 175 + \frac{45}{50} \times 20$$

$$= 175 + 18 = 193 \text{ Ans.}$$

Ques :- Calculate mean from the shortcut method.

Mid values (x)	F	dx	fdx
5	4	-30	-120
15	8	-20	-160
25	11	-10	-110
<u>35</u>	15	0	0
45	11	20	110

55	7	20	140
65	4	30	120
	60		20

$$\bar{X} = A + \frac{\sum f d_n}{\sum f} = 35 - \frac{20}{60} = 35 - 0.33$$

$$= 34.67 \text{ dm.}$$

Ques:- Calculate A.M. by shortcut and step deviation method.

L-I	f	X	d _n	f d _n
0-10	5	5	-87.5	-187.5
10-20	10	17.5	-25	-250
25-35	16	30	-12.5	-375
35-50	30	42.5	0	0
50-60	19	59.5	17	323
70-78	10	74	31.5	315
78-85	5	81.5	39	195
85-100	5	92.5	50	250
	100			270.5

$$\text{Ans } \bar{X} = A + \frac{\sum f d_n}{\sum f} = 42.5 + \frac{270.5}{100} = 42.5 + 2.705$$

$$= 45.205$$

Ques:- Wages in ₹

Wages in ₹	F
less than 10	30
" " 20	70
20 - 30	50
20 - 40	98
40 and above	332
50 " "	308
60 - 70	132
70 and above	14

wages in ₹	F	CF	F
less than 10	30	0-10	30
" " 20	70	10-20	40
20-30	50	20-30	50
20-40	98	30-40	48
40 and above	332	40-50	24
50 and above	308	50-60	162
60-70	132	60-70	132
70 and above	14	70-80	14

Ques:- Calculate mean from the following distribution -

C-I	F	X	FX
0-10	5	5	25
10-25	10	17.5	175
25-35	16	30	480
35-50	30	42.5	1275
50-69	19	59.5	1130.5
70-78	10	74	740
78-85	5	81.5	407.5
85-100	5	92.5	462.5
	100		4695.5

$$\bar{X} = \frac{\sum FX}{\sum F} = \frac{4695.5}{100}$$

$$\bar{X} = 46.95 \text{ Am}$$

Ques:- Find the A.M. by shortcut method.

C-I	F	C-I	X	du	fdn
below-4	3	0-4	2	-6	-18
4-12	7	4-12	8	0	0
12-24	15	12-24	18	10	150
24 and above	20	24-40	32	24	480
	45				612

$$\bar{x} = \frac{A + \sum fd'u}{\sum f} = \frac{8 + 612}{45} = 8 + 13.6$$

$$= 21.6 \text{ Ans}$$

Ques^o:- Find mean from the three-method.

Class	F	C-T
Below-10	5	C-T
10-20	10	0-10
20-30	20	10-20
30-40	40	20-30
40-50	30	30-40
50-60	20	40-50
60-70	10	50-60
70 and above	4	60-70

C-T	F	X	FX	du	fd'u	d'u = $\frac{du}{10}$	f d'u
0-10	5	5	25	-30	-150	-3	-15
10-20	10	15	150	-20	-200	-2	-20
20-30	20	25	500	-10	-200	-1	-20
30-40	40	35	1400	0	0	0	0
40-50	30	45	1350	10	300	1	30
50-60	20	55	1100	20	400	2	40
60-70	10	65	650	30	300	3	30
70-80	4	75	300	40	160	4	16
	139		5475		610		61

$$i) \bar{X} = \frac{\sum fX}{\sum f} = \frac{5475}{139} = 39.388$$

$$ii) \bar{X} = A + \frac{\sum fdm}{\sum f} = 35 + \frac{610}{139} = 39.388$$

$$iii) \bar{X} = A + \frac{\sum fd'u \times i}{\sum f} = 35 + \frac{61 \times 10}{139} = 39.388$$

Ques:- And the A.M. from step-deviation method.

below-20	3
20-30	5
30-40	8
40-50	5
50 and above	4

C-I	f	X	dm	d'u	fd'u
10-20	3	15	-20	-2	-6
20-30	5	25	-10	-1	-5
30-40	8	35	0	0	0
40-50	5	45	10	1	5
50-60	4	55	20	2	8
	<u>25</u>				<u>2</u>

$$\bar{X} = A + \frac{\sum fd'u \times i}{\sum f} = 35 + \frac{2 \times 10}{25}$$

$$= 35 + \frac{20}{25} = 35.8 \text{ Ans.}$$

Q To find unknown frequency or value of the variable.

C-I	F	X	CF	FX
60-62	15	61	15	915
63-65	54	64	69	3456
66-68	A	67	69+A	67A
69-71	21	70	150+A	5670
72-74	27	73	177+A	1971
	177+A			12012+67A

$$\text{Mean} = 67.45$$

$$\bar{x} = \frac{\sum FX}{\sum F}$$

$$67.45 = \frac{12012 + 67A}{177 + A}$$

$$67.45(177 + A) = 12012 + 67A$$

$$67.45A + 11938.65 = 12012 + 67A$$

$$0.45A = 73.35$$

$$A = \frac{73.35}{0.45} = 163$$

Ques:-

X	F	FX
20	8	160
30	12	360
A	20	20A
50	10	500
60	6	360
70	4	280
	60	1660+20A

Mean = 41

$$41 = \frac{1660 + 20A}{60}$$

$$2460 = 1660 + 20A$$

$$800 = 20A$$

$$A = 40 \text{ km}$$

Ques:-

C-I	No. of Shop	X	FX
0-10	12	5	60
10-20	18	15	270
20-30	27	25	675
30-40	A	35	35A
40-50	17	45	765
50-60	6	55	330
	80+A		2100+35A

Mean = 28

$$28 = \frac{2100 + 35A}{80 + A}$$

$$2240 + 28A = 2100 + 35A$$

$$140 = 7A$$

$$A = 20 \text{ km}$$

Ques :- By using mean = 16.41 calculate missing frequency.

C-I	F	X	F	
0-5	3	2.5	7.5	Mean = 16.41
5-10	4	7.5	30	$16.41 = \frac{507.5 + 12.5A}{29 + A}$
10-15	A	12.5	12.5A	
15-20	10	17.5	175	$475.89 + 16.41A = 507.5 + 12.5A$
20-25	7	22.5	157.5	$3.91A = 31.61$
25-30	5	27.5	137.5	$A = 8.08, \text{Ans}$
	29+A		507.5+12.5A	

Median

Median no. = $\frac{N+1}{2}$ In Individual series.

Median no. = $\frac{N}{2}$ In Continuous series.

$$\text{Median} = L_1 + \frac{i}{f} \left(\frac{n}{2} - C \right)$$

~~Median~~

<u>Ques</u>	Marks	F	CF	
	0-7	19	19	Median no = $\frac{N}{2}$
	7-14	25	44	
	14-21	36	80	" " = $\frac{274}{2}$
	21-28	72	152	
	28-35	51	203	Median no. = 137 Item
	35-42	43	246	
	42-49	28	274	Median = $l_1 + \frac{f}{F} \left(\frac{n}{2} - C \right)$
		274		

<u>Ques</u>	C-I	F	CF	
	0-10	12	12	= $\frac{21+7}{72} (57)$
	10-20	18	30	
	20-30	27	57	= $\frac{21+399}{72}$
	30-40	20	77	= $\frac{21+5.54}{72}$
	40-50	17	94	
	50-60	6	100	Median = 26.54
		100		

$$\text{Median no.} = \frac{100}{2} = 50$$

$$\text{Median} = 20 + \frac{10}{27} (50 - 30)$$

$$= 20 + \frac{10 \times 20}{27} = 20 + \frac{200}{27}$$

$$= 20 + 7.40$$

$$= 27.40 \text{ Ans.}$$

Ques:- Find out missing frequency if median is 52.

C-I	F	CF	Median = 52.
0-10	5	5	$52 = 50 + 10 \left(\frac{72+A-32+A}{2} \right)$
10-20	8	13	
20-30	7	20	$52 = 50 + 10 \left(\frac{72+A-64-2A}{2} \right)$
30-40	12	32	
40-50	A	32+A	$52 = 50 + 10 \left(\frac{8-A}{2} \right)$
50-60	20	52+A	
60-70	10	62+A	$52 = 50 + \frac{8-A}{4}$
70-80	10	72+A	
	72+A		$2 = \frac{8-A}{4}$

$$8 = 8 - A$$

$$\boxed{A = 0} \text{ Ans.}$$

Ques:- From the following data, compute median.

C.I.	f	C.f.	
0-25	30	30	$M. no. = \frac{N}{2} f = \frac{200}{2} = 100$
25-50	50	80	
50-75	80	160	$M = 50 + \frac{25}{80} (100 - 80)$
75-100	40	200	
	200		$= 50 + \frac{25}{80} (80)$
			$= 50 + 6.25 = 56.25 \text{ km}$

Ques: Find the median from the following data-

S.NO.	f	asc.	
1	745	465	$M. no. = \frac{N}{2} = \frac{10+11}{2} = 5.5$
2	465	595	
3	956	650	Median = $\frac{5^{\text{th}} \text{ observation} + 6^{\text{th}}}{2}$
4	750	745	
5	1050	750	$= \frac{750 + 870}{2}$
6	650	870	
7	900	900	$= \frac{1620}{2} = 810 \text{ gr}$
8	870	956	
9	575	1050	
10	1250	1250	

Ques, Find out median from the following series-

mid values	f	C.I.	C.f.	M. no = $\frac{N}{2} = \frac{250}{2}$
5	15	0-10	15	$= 125$
15	20	10-20	35	
25	25	20-30	60	Median = $L + \frac{f}{f_1 - f_0} \left(\frac{N}{2} - C \right)$
35	24	30-40	84	
45	12	40-50	96	$= 50 + 10 \left(\frac{125 - 96}{31} \right)$
55	31	50-60	127	
65	71	60-70	198	$= 50 + 10 \left(\frac{125 - 96}{31} \right)$
75	52	70-80	250	
	250			$= 50 + 9.35$
				$= 59.35$ <u>Ans.</u>

Ques: Find the median from the following distribution -

C-I	f	C.f.	M. no. = $\frac{73}{2} = 36.5$ Item.
0-4	3	3	
4-8	9	12	Median = $12 + \frac{f}{f_1 - f_0} \left(\frac{N}{2} - C \right)$
8-12	18	30	$= 12 + \frac{1}{5} \times 6.5$
12-16	20	50	
16-20	16	66	
20-24	7	73	$= 12 + 1.3 = 13.3$ <u>Ans.</u>

* Mode :-

- Mode In Individual series -

Ex:- 5, 8, 10, 11, 15, 18, 20, 22, 25, 30.

Sol:- No mode.

Ques:- 4, 8, 10, 12, 13, 10, 15, 12, 13, 12
Mode = 12.

- Mode In continuous series -

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Ques	C-I	f	Mode
	10-20	10	$= \text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$ $= 35 + \frac{105 - 70}{2 \times 105 - 70 - 74} \times 5$ $= 35 + \frac{35}{144} \times 5$ $= 35 + \frac{35 \times 5}{66}$ $= 35 + 2.675 = 37.65$
	20-25	35	
	25-30	45	
	30-35	70	
Mode class	35-40	105	
	40-45	74	
	45-50	51	
	40-50	30	

Ques 1:- Calculation of mode in Inclusive series -

C-I	f	C-I.	Mode = $43.5 + \frac{20-65}{2 \times 20-65-51} \times 4$
32-35	10	31.5-35.5	
36-39	37	35.5-39.5	= $43.5 + \frac{15}{160-114} \times 4$
40-43	65	39.5-43.5	
44-47	100	43.5-47.5	= $43.5 + \frac{60}{46}$
48-51	51	47.5-51.5	
52-55	35	51.5-55.5	= $43.5 + 1.36$
56-59	18	55.5-59.5	= 44.86 Ans
60-63	4	59.5-63.5	

Ques 2:- Calculate mode -

X	58	60	61	62	63	64	65	66	68	70
f	4	6	5	10	20	22	24	6	2	1

Mode = 65

Ques 3:- Find mode -

no. of days	no. of Students (f)
0-5	29
5-10	195
10-15	241
15-20	117
20-25	52
25-30	10

30-35 6

35-40 3

40-45 2

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 10 + \frac{241 - 195}{2 \times 241 - 195 - 117} \times 5$$

$$= 10 + \frac{46}{482 - 312} \times 5$$

$$= 10 + \frac{46}{170} \times 5$$

$$\text{Mode} = 10 + 1.335 = 11.35$$

Ques. Find mode -

C-I	f
0-10	5
10-20	15
20-30	20
30-40	20
40-50	32
50-60	14
60-70	11
70-80	5

$$\text{Mode} = 40 + \frac{32 - 20}{2 \times 32 - 20 - 14} \times 10$$

$$= 40 + \frac{12}{64 - 34} \times 10$$

$$= 40 + \frac{12}{30} \times 10$$

$$= 40 + 4 = 44$$

* Mode in unequal distribution -

C-I	f	C-I	f	Mode = $20 + \frac{12-10}{2 \times 12 - 10 - 7} \times 10$
0-5	5	0-10	9	
5-10	4	10-20	10	$= 20 + \frac{2}{24-17} \times 10$
10-20	10	20-30	12	$= 20 + \frac{2}{7} \times 10$
20-22	3	30-40	7	
22-26	4	40-50	7	$= 20 + 2.85$
26-30	5	50-60	5	$= 22.85$ <u>Ans</u>
30-40	7			
40-50	7			
50-55	3			
55-60	2			

Ques: Mode in Inclusive series -

C-I	f	C-I	Mode = $43.5 + \frac{80-65}{2 \times 80 - 65 - 51} \times 4$
32-35	10	31.5-35.5	
36-39	37	35.5-39.5	$= 43.5 + \frac{15}{160-116} \times 4$
40-43	65	39.5-43.5	$= 43.5 + 1.4$
44-47	80	43.5-47.5	$= 44.9$ <u>Ans</u>
48-51	51	47.5-51.5	
52-55	35	51.5-55.5	
56-59	18	55.5-59.5	
60-63	4	59.5-63.5	

* Determination of mode from mean and median -

$$\text{Mode} = 3\text{median} - 2\text{mean}$$

Ques:- If mean is 26, mode is 28.5, find median.

$$\text{Mode} = 3\text{median} - 2\text{mean}$$

$$28.5 = 3M - 2 \times 26$$

$$M = \frac{28.5 + 52}{3}$$

$$M = \frac{80.5}{3} = 26.83 \quad \underline{\text{Ans}}$$

Ques:- Mode = 32.1, median = 35.4, find mean.

$$\text{Mode} = 3\text{Median} - 2\text{Mean}$$

$$32.1 = 3 \times 35.4 - 2M$$

$$32.1 = 106.2 - 2M$$

$$M = \frac{74.1}{2} = 37.05 \quad \underline{\text{Ans}}$$

Ques:- Find out mode from the following data -

$$\text{Mode} = 40 + \frac{32 - 20}{2 \times 32 - 20 - 14} \times 10$$

$$= 40 + \frac{12}{64 - 34} \times 10 = 40 + \frac{12}{30} \times 10$$

$$= 40 + 4 = 44 \quad \underline{\text{Ans}}$$

C-I	f
0-10	5
10-20	15
20-30	20
30-40	20 f_0
40-50	32 f_1
50-60	14 f_2
60-70	14
70-80	5

Ques:- Find mean, median, mode from the following data -

C-I	f	u	fu	C-f
10-20	5	15	75	5
20-30	8	25	200	13
30-40	20	35	700	33
40-50	12	45	540	45
50-60	15	55	825	60
60-70	20	65	1300	80
70-80	14	75	1050	94
80-90	10	85	850	104
	104		5540	

$$\text{Mean} = \frac{\sum fu}{\sum f} = \frac{5540}{104} = 53.26$$

$$\text{M. no.} = \frac{N}{2} = \frac{104}{2} = 52$$

$$\text{Median} = l_1 + \frac{\frac{N}{2} - C}{f} (l_2 - l_1)$$

$$= 50 + \frac{52 - 45}{15} (50 - 40)$$

$$= 50 + \frac{10}{15} \times 7 = 50 + 4.66 = 54.66$$

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Ques:- Find mode from the following data -

C-I	f	C-I	f
0-2	4	0-6	12
2-6	8	6-12	24
6-9	10	12-18	36
9-12	14	18-24	38
12-16	16	24-30	37
16-18	20	30-36	6

$$\text{Mode} = 18 + \frac{38 - 36}{2 \times 38 - 36 - 37} \times 6$$

$$= 18 + \frac{2}{76 - 73} \times 6$$

$$= 18 + \frac{2 \times 6}{3}$$

$$= 18 + 4 = 22$$

18-20 24

20-24 14

24-26 16

26-28 18

28-30 10

30-36 6

Ques:- Find out median and mode from the following data.

C-I	F
0-9	2
10-19	10
20-29	18
30-39	20
40-49	30
50-59	25
60-69	16
70-79	10
80-89	8
90-99	3
	150

Ques: In a series mean and median are 10 and 11. Find out mode.

$$\text{Mode} = 3 \times 11 - 2 \times 10 = 33 - 20 = 13 \text{ Ans.}$$

Ques: (i) Find out median if mean = 20 and mode = 10.

(ii) Median and mean are 24 and 26. Find mode.

(iii) Mean is 26 and mode is 28.5. Find median.

(i) $\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$

$$10 = 3M - 2 \times 20$$

$$10 = 3M - 40$$

$$50 = 3M$$

$$\text{Median} = \frac{50}{3} = 16.67 \text{ Ans.}$$

(ii) $\text{Mode} = 3 \times 24 - 2 \times 26 = 72 - 52 = 20 \text{ Ans.}$

(iii) $\text{Mode} = 3 \times \text{Median} - 2 \times 26$

$$28.5 = 3M - 52$$

$$3M = 80.5 \Rightarrow \text{median} = 26.83 \text{ Ans.}$$

★ Missing frequency in mode :-

Median and mode of data is 27 and 26.

C.I	f	CF
0-10	3	3
10-20	A	11
20-30	20	31
30-40	12	43
40-50	B	43+B
	43+B	

Median = 27 and mode = 26

is lies in class 20-30

Hence it is median class as well as mode class.

$$\text{Mode} = 20 + \frac{20+A}{2 \times 20 - A - 12} \times 10$$

$$26 = 20 + \frac{20+A}{40-A-12} \times 10$$

$$6 = \frac{200-10A}{18-A}$$

$$168 - 6A = 200 - 10A$$

$$4A = 32$$

$$\boxed{A = 8}$$

$$\text{Median no.} = \frac{N}{2} = \frac{43+B}{2}$$

$$\text{Median} = 20 + \frac{10}{20} \left(\frac{43+B}{2} - 11 \right)$$

$$27 = 20 + \frac{1}{2} \left(\frac{43+B}{2} - 22 \right)$$

$$7 \times 4 = 21 \times B$$

$$B = 28 - 21$$

$$\boxed{B = 7}$$

Ques: Find out missing frequency if mode = 24.

C.I.	f
0-10	14
10-20	A
20-30	27
30-40	B
40-50	15
	100

$$\text{Mode} = 4 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$24 = 20 + \frac{27 - A}{2 \times 27 - A - B} \times 10$$

$$4 = \frac{270 - 10A}{54 - A - B}$$

$$216 - 4A - 4B = 270 - 10A$$

$$6A - 4B = 54$$

$$3A - 2B = 27 \quad (1)$$

$$14 + 27 + 15 + A + B = 100$$

$$A + B = 44 \quad (2)$$

from eqn (1) and (2)

$$3A - 2B = 27$$

$$2A + 2B = 88$$

$$5A = 115$$

$$\boxed{A = 23}$$

$$A + B = 44$$

$$23 + B = 44$$

$$\boxed{B = 21}$$

Ques:- obtain mean, median and mode from the following data:

C-I	f	u	f.u.	C.f.
0-10	13	5	65	13
10-20	22	15	330	35
20-30	30	25	750	65
30-40	20	35	700	85
40-50	15	45	675	100
	100		2520	

$$\text{Mean} = \frac{\sum fu}{\sum f} = \frac{2520}{100}$$

$$= 25.2$$

$$\text{Median no.} = \frac{N}{2} = \frac{100}{2}$$

$$= 50$$

$$\text{Median} = 20 + 10 \left(\frac{50 - 35}{30} \right)$$

$$= 20 + \frac{1}{3} (15)$$

$$= 20 + 5 = 25$$

$$\text{Mode} = 20 + 30 - 22 \times 10$$

$$60 - 22 - 20$$

$$= 20 + \frac{8 \times 10}{10}$$

$$= 20 + 4.44 = 24.44$$

Ques:- find out missing frequency when median = 25 and mode = 25.55.

C-I	f	C.f.
0-10	5	5
10-20	A	5+A
20-30	20	25+A
30-40	B	25+A+B
40-50	3	28+A+B

$$\text{Mode} = 20 + 30 - 22$$

$$20 + \frac{20 - A}{40 - A - B} \times 10$$

$$25.55 = 20 + \frac{200 - 10A}{40 - A - B}$$

$$40 - A - B$$

$$5.55(40 - A - B) = 200 - 10A$$

$$222 - 5.55A - 5.55B = 200 - 10A$$

$$5.55A - 5.55B = -22 - 10A$$

$$4.55A - 5.55B = -22 \quad \text{--- (1)}$$

$$\text{M. no.} = \frac{N}{2} = \frac{28 + A + B}{2}$$

$$\text{Median} = 20 + \frac{10}{20} \left(\frac{28 + A + B}{2} - 5 - A \right)$$

$$25 = 20 + \frac{1}{2} \left(\frac{28 + A + B - 10 - 2A}{2} \right)$$

$$5 \times 4 = -A + B + 18$$

$$20 = -A + B + 18$$

$$A - B = -2 \quad \text{--- (2)}$$

From eqn (1) and (2)

$$A = 9.9 = 10$$

$$B = 11.9 = 12$$

Quest Mode and median are 33.5 and 34.

C-I	f
0-10	4
10-20	16
20-30	A
30-40	B
40-50	C
50-60	6
60-70	4

★ Geometric mean -

$$G.M = N \sqrt{(u_1) \times (u_2) \times (u_3) \dots}$$

Ques:- Find geometric mean of - 3, 27

$$\begin{aligned} G.M. &= \sqrt[2]{3 \times 27} \\ &= \sqrt{3 \times 3 \times 3 \times 3} \\ &= 9 \text{ Ans.} \end{aligned}$$

Ques:- Find geometric mean of - 4, 8, 16

$$\begin{aligned} G.M. &= \sqrt[3]{4 \times 8 \times 16} \\ &= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \\ &= 8 \end{aligned}$$

16, 25, 36

$$\begin{aligned} G.M. &= \sqrt[3]{16 \times 25 \times 36} \\ &= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 2 \times 2 \times 3 \times 3} \\ &= 2 \times 2 \sqrt[3]{5 \times 5 \times 3 \times 3} \\ &= 4 \sqrt[3]{225} \text{ Ans.} \end{aligned}$$

★ Uses of geometric mean -

- 1- It is the most useful in the study of social and economical phenomena, where small number are to be given importance.

2- If some items of series are very large in magnitude and others are small then geometric mean is the most representative average.

3- The most important use of geometric mean in case of index number, Fisher Ideal Index number is based on geometric mean.

* Harmonic mean -

$$H.M. = \frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots}$$

Ques:- Calculate harmonic mean from the following data - 2, 4, 5, 6.

$$\begin{aligned} H.M. &= \frac{4}{\frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}} \\ &= \frac{4}{\frac{30+15+12+10}{60}} = \frac{240}{67} = 3.58 \end{aligned}$$

* Merits of Harmonic Mean -

- 1- Its value is defined and proper.
- 2- It is based on the observation.

- 3- It gives highest weight to smallest unit. By this larger unit do not influence its value.
- 4- It is specially used in finding average of rate or times.
- 5- Its algebraic treatment is possible.

* Demerits of Harmonic mean :-

- 1- It is used in specific case only.
- 2- It is not easy to understand and involve complicated calculations.
- 3- It cannot be calculated if the value of any item is unknown.
It may not present in the series.

* Relationship between A.M., G.M. and H.M.

(i) In case of series are ~~individual~~ identical -

$$A.M. = G.M. = H.M.$$

(ii) If the items are not equal -

$$AM \geq GM \geq HM$$

(iii) If there are two terms -

$$GM = \sqrt{AM \times HM}$$

$$HM = \sqrt{AM \times GM}$$

$$AM = \sqrt{GM \times HM}$$