UNIT – 5
Job Sequencing

* Job Sequencing :- The selection of an appropriate order in which to service waiting customers is called sequencing.

A general sequencing problem may be defined as follows -

Let there are n jobs (1, 2, 3 --- n) which have to be processed one at a time at each of m machine A, B, C ---. The order of the machine is given for each job in which it should go to the machine A, B, C ---. The order of the machine is given. The time required by the jobs on each of the machine is also given. Then the problem is to find the sequence out of $(n!)^m$ sequences, which optimizes (minimize) the total time elapsed from the start of the first job to the completion of the last job.

Mathematically,

Let A_i = time required for job i on machine A.

B_i = time required for job i on machine B.

T = total elapsed time for jobs 1, 2, 3 ---- n i.e. the time from start of the first job to completion of the last job.

The problem is to determine a sequence $(i_1, i_2 \ldots i_n)$ where $(i_1, i_2 \ldots i_n)$ is the permutation of integers which will minimize T.

Analytic method have been developed for solving only four simple cases.

1. n jobs and 2 machines A and B. All jobs

processed in the order AB.

2- n jobs and 3 machines A, B and C. All jobs processed in the order ABC.

3- n jobs and m machines A, B, C.... M. All jobs processed in the order ABC.... M.

4- 2 jobs and m machines. Each job to be processed through the machine in a prescribed order.

* Total elapsed time :- This is the time between starting the first job and completing the last job. It is denoted by T.

* Idle time on a machine :- This is the time for which a machine remains idle during the total elapsed time.

* Processing time :- It means the time required by each job on each machine.

* Processing order :- The order in which various machines are required for completing the job.

* <u>Sequencing Decision Problem for n-job on two machines (Johnson's Method)</u> :-

Here we consider the problem of processing n-jobs (1, 2, 3 ---- n) on two machines A and B under the following assumptions -

1- Each job is processed in the order AB.

2- A_i = Processing time of i^{th} job on machine A. $(i = 1, 2, 3 .... n)$

3- B_i = Processing time of i^{th} job on machine B. $(i = 1, 2, 3 .... n)$

The problem is to find the sequence of jobs to be performed on two completion of the last job is minimize.
The procedure for the solution of the above problem was developed by Johnson and Bellman. The method is based on minimizing the idle time for second machine. The Johnson's procedure for determine an optimal sequence is a follows-

Step 1- Examine the A_i and B_i for $i=1,2,3 \ldots n$ and select the minimum of these. If there are two or more minimum processing time then select any one of them arbitrarily.

Step 2- If the minimum processing time is for machine A, process that job and place it at the beginning of the sequence.
If the minimum processing time is for machine B, process job first and place it at the end of the sequence.

Step 3- Cross all the jobs already assign and repeat step 1 and 2, placing the remaining jobs next to first or next to last, until all the jobs have been assign.

Step 4- Calculate the time at which each job in the sequence will be processed on machine A. This time can be calculated as follows-
Time at which i^{th} job in a sequence finish on machine A = time when the $(i-1)^{th}$ job in a sequence finish on machine A and the time for start of first job on machine A is zero.

Step 5- Calculate the time at which each job in the sequence will start and finish on machine B as follows –

(i) Time when first job in a sequence start on machine B = time when the first job in a sequence finish on machine A.

(ii) Time the i^{th} job in the sequence finish at B = time when the i^{th} job in a sequence start on machine B + the processing time of i^{th} job on machine B. ($i = 1, 2, 3 --- n$).

(iii) Time at which the $(i+1)^{th}$ job in a sequence finish on machine B = maximum time when the $(i+1)^{th}$ job in a sequence finish on the machine at the time when the i^{th} job in a sequence finish on machine B. ($i = 1, 2, 3 --- n$).

Step 6 – Calculate the total elapsed time to process all jobs through two machines i.e., when the n^{th} job in a sequence finish on machine B.

Step 7 – Compute the idle time for machine A and B.

Idle time for machine A = time for n^{th} job in a sequence finish on machine B.

Idle time for machine B = time at which the first job in a sequence finish on machine A + Time i^{th} job in a sequence start on machine B – Time when $(i-1)^{th}$ job in a sequence finish on machine B.

Ques- A company has 3 jobs on hand. Each of these must be processed from two departments. The sequential order for which is-

Department A - Press shop
Department B - Finishing

The table below list the number of days required by each job in each department-

	Job I	Job II	Job III
Department A →	8	6	5
Department B →	8	3	4

Find the sequence in which the 3 jobs should be processed so as to take minimum time to finish all the 3 jobs.

Sol:- Smallest time in the given process-

I	III	II

Jobs	Department A		Department B		Idel time of B
	Time in	Time out	Time in	Time out	
I	0	8	8	16	8
III	8	13	16	20	
II	13	$\boxed{19}$	20	$\boxed{23}$	

Total minimum elapsed time = 23 days
Ideal time of department A = 23 − 19
= 4 days.

* Travelling salesman problem :- Travelling salesman problem is very similar to the assignment problem except that in the formal there is an additional restrictions that a salesman who start

from his home city, visit each city only once and return to his home city. The problem is to find to routes shortest in distance (time or cost). If the number of cities is 3 (A, B and C) of which starting base A there are two possible routes $A \to B \to C$ and $A \to C \to B$. In general for n cities there are $(n-1)!$ possible routes.

Mathematically the problem may be stated as follows-

$$\text{Minimize } \sum_i \sum_j c_{ij} x_{ij} \quad *$$

Ques- There are 7 jobs, each of which has to go through the machine A and B in the order AB. Processing time hours are given as-

Job	1	2	3	4	5	6	7
Machine A	3	12	15	6	10	11	9
Machine B	8	10	10	6	12	1	3

Determine a sequence of these jobs that will minimize the total elapsed time T.

Sol:- We obtain the sequence

1	5	2	3	4	7	6

	Machine A		Machine B		Idal time of
Job	Time in	Time out	Time in	Time out	machine B
1	0	3	3	11	3
4	3	9	11	17	0
5	9	19	19	31	2
3	19	34	34	44	3
2	34	46	46	56	2
7	46	55	56	59	0
6	55	$\boxed{66}$	66	$\boxed{67}$	7

Total elapsed time = 67
Ideal time = 67 − 66 = 1 hour

Ques –

Job	1	2	3	4	5	6
Machine A	30	120	50	20	90	110
Machine B	80	100	90	60	30	10

Sol:- Sequence

4	1	3	2	5	6

	Machine A		Machine B		
Job	Time in	Time out	Time in	Time out	Ideal time
4	0	20	20	80	20 (B)
1	20	50	80	160	0
3	50	100	160	250	0
2	100	220	250	350	0
5	220	310	350	380	0
6	310	$\boxed{420}$	380	$\boxed{430}$ ~~390~~	40

Total elapsed time = 430 hrs
Ideal time for A = 430 − 420 = 10 hrs
" " " B = 20 + 40 = 60

*. Subject to the additional constraint that x_{ij} is to be so chosen that no city is visited twice before the tour of all the cities is completed.
The travelling salesman problem can be written in the form of square assignment problem.

	1	2	3	----	n
1	∞	C_{12}	C_{13}	----	C_{1n}
2	C_{21}	∞	C_{23}	----	C_{2n}
Form 3	C_{31}	C_{32}	∞	----	C_{3n}
⋮	⋮			∞	
n	C_{n1}	C_{n2}	----	----	∞

Ques- Solve the following salesman problem –

∞	4	10	14	2
12	∞	6	10	4
16	14	∞	8	14
24	8	12	∞	10
2	16	4	16	∞

∞	2	8	12	0
8	∞	2	6	0
8	6	∞	0	6
16	0	4	∞	2
0	4	2	14	∞

	A	B	C	D	E
A	∞	2	6	12	$\boxed{0}$
B	8	∞	$\boxed{0}$	6	$\cancel{0}$
C	8	6	∞	$\boxed{0}$	6
D	16	$\boxed{0}$	2	∞	2
E	$\boxed{0}$	4	$\cancel{0}$	14	∞

It is a feasible solution and re-check salesman conditions.

$A \to E \to A$ condition is not satisfied.

General rule:

1- If there is no option then select the next lowest cost in the matrix.

2- First reference must be to 0.

3- After 0 the next preference is to 1 or 2.
4- If there is no option then it may be high value.

	A	B	C	D	E
A	∞	$\boxed{2}$	6	12	~~0~~
B	8	∞	$\boxed{0}$	6	~~0~~
C	8	6	∞	$\boxed{0}$	6
D	16	~~0~~	~~0~~	∞	$\boxed{2}$
E	$\boxed{0}$	4	~~0~~	14	∞

$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A$

★ **Money value :-** Money value is the value of money to be spent after the period, on the original value with some interest.
The value of money change with time. Since has value over time. This can be explain with the following example-

Ex- If we borrow ₹100 at the rate of interest 10% per year and spend this amount today, then we have to pay ₹110 after one year.

★ **Present value / Present Worth Factor :-**
Present value is the current value of money or the running cost of an item that deteriorates over a period of time.
If r is the rate of interest then $\frac{1}{(1+r)^n}$ is called the present value factor of one rupee spent in n years.

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★ Discount rate (Depreciation value) :-
Discount rate refers to the interest rate used to determine the present value of future cash flows. It is not just the time value of money but the risk or uncertainity of future cash flows.

$$v = \frac{1}{(1+r)}$$