UNIT - 4

Inventory Theory

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* **Inventory :-** Inventory is a stock of items which is to be used in future. So in another words the inventory is also called stock.
An inventory consist of a usable, but idle resources such as money, machines and men.

• **Inventory Costs** - The three costs considered inventory control models are -

1- Inventory carrying cost or stock holding costs.
2- Storage costs.
3- Procurement costs or set-up costs.
4- Total inventory cost.

* **Inventory carrying costs or Stock holding costs :-**
Some components of the stock holding cost are -

1- Cost of money or capital tied up in inventories - More borrowed from the banks may cost interest of about 18%. It is generally taken somewhere around 15% to 20% of the value of the inventories.

2- Cost of Storage space - This consist of rent for space. Besides space expenses, this will also include heating, lighting and other atmospheric control expenses. Typical values may vary from 1 to 3%.

3- Handling costs - Expenditure on stock holding is called handling costs. Such as cost of labour, overhead cranes, gantries and other machinery used for this purpose.

✓ ★ Ordering or Set-up cost :- When an item is produced internally, ordering cost is referred as set-up cost which includes both paper work costs and physical preparation costs.

Ordering cost = (Cost per order or per set-up) × (Number of orders or setups places in the planning period).

✓ ★ Shortage or stock-out and customer-service cost :- The shortage of items occurs when items cannot be supplied on demand. Therefore, shortage costs are usually interpreted in two ways-

1- The supply of items is awaited by the customers i.e., the items are back ordered.

2- Customers are not ready to wait.

This situation may lead to less of customer goodwill and therefore causes loss of sale.

Therefore,

Shortage cost = (cost of being short one unit of an item) × (average number of units cost)

The average number of units short in a planning period is obtain by average number of units short

$$= \left(\frac{\text{minimum shortage} + \text{maximum shortage}}{2}\right) \times \text{period of shortage}$$

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★ Inventory Control Problem :- The inventory control problem consist of determination of two basic factors-

1- When to order - This is related to the lead time of an item.
There should be sufficient stock for each item so that customers, order can be reasonably met from this stock until replenishment. This stock level known as reorder level. It is obtain by compromising the cost of maintaining these stocks and the dis-service to the customer if his orders are not filled in time.

2- How much to order - We know each order is related with its ordering cost. To maintain it low, the number of orders should be as few as possible. But large order size would imply high inventory carrying cost. The over problem is determine how must order is solved by compromising.

★ Concept of Economic Ordering Quantity (E.O.Q.) :-
The economic ordering quantity is that size of order which minimizes total annual cost of carrying inventory and cost of ordering under the assumed conditions of certainly and that annual demands are known.

Ques 1- Define the following :-
(i) Transportation problem
(ii) Mathematical formulation of Assignment problem.
(iii) Unbalanced assignment problem.

Sol :- (i) Transportation Problem :-
Transportation problem is a special kind of linear programming problem in which goods are transported from a set of sources to a set of destination subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized.

Let m = the number of sources.
n = the number of destinations.
a_i = the supply at the source i.
b_j = the demand at the destination j.
c_{ij} = the cost of transportation per unit from ith source to jth destination.
x_{ij} = the number of units to be transported from the source ith to the jth destination.

(ii) Mathematical formulation of Assignment Problem :-
Mathematically, the assignment problem can be stated as -
Minimize the total cost:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

where $i = 1, 2, 3 \ldots\ldots n$
$j = 1, 2, 3 \ldots\ldots n$

Subject to restrictions

$$x_{ij} = \begin{cases} 1, & \text{if ith person is assigned jth job} \\ 0, & \text{if not.} \end{cases}$$

$\sum_{j=1}^{n} x_{ij} = 1$ (one job is done by the ith person $i = 1, 2, 3 \ldots n$)

$\sum_{i=1}^{n} x_{ij} = 1$ (only one person should be assigned the jth job, $j = 1, 2, 3 \ldots n$)

where x_{ij} denotes the jth job is to be assigned to the ith person.

(iii) **Unbalanced Assignment Problem :-**

If the cost matrix of an assignment problem is not square matrix, the assignment problem is called unbalanced assignment problem. In such cases add dummy rows or columns with zero costs. Then the usual assignment algorithm can be applied to this resulting balanced problem.

Example —

Job

Person	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24

Job

Person	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24
D (dummy)	0	0	0	0

Ques 2:- Solve the following transportation problem by using -
(i) Matrix minima method
(ii) VAM
(iii) Test for optimality.

Destination

		D_1	D_2	D_3	D_4	Supply
	I	1	2	1	4	30
Origin	II	3	3	2	1	50
	III	4	2	5	9	20
Demand		20	40	30	10	100

Sol:- (i) By matrix minima method-

Destination

		D_1	D_2	D_3	D_4	Supply
	I	$\boxed{20}$ 1	2	$\boxed{10}$ 1	4	30/10
Origin	II	3	$\boxed{20}$ 3	$\boxed{20}$ 2	$\boxed{10}$ 1	50/40/20/0
	III	4	$\boxed{20}$ 2	5	9	20/0
Demand		20/0	40/20/0	30/20/0	10/0	100

Minimum cost = 1×20 + 1×10 + 3×20 + 2×20 + 1×10 + 2×20
= 20 + 10 + 60 + 40 + 10 + 40
= 180 Rs.

Initial Basic Feasable Solution -

$x_{11} = 20$

$x_{13} = 10$

$x_{22} = 20$

$x_{23} = 20$

$x_{24} = 10$

$x_{32} = 20$

(ii) By VAM –

	D_1	D_2	D_3	D_4	Supply	Penalty
I	1	2	1	4	30	1
II	3	3	2	$\boxed{10}$ 1	50/40	1
III	4	2	5	9	20	1
Demand	20	40	30	10/0	100	
Penalty	2	1	1	3 (max)		

	D_1	D_2	D_3	Supply	Penalty
I	$\boxed{20}$ 1	2	1	30/10	1
II	3	3	2	40	1
III	4	2	5	20	1
Demand	20/0	40	30	90	
Penalty	2 (max)	1	1		

	D_2	D_3	Supply	Penalty
I	2	$\boxed{10}$ 1	10/0	1 ← (max)
II	3	2	40	1
III	2	5	20	1
Demand	40	30/20	70	
Penalty	1	1		

	D_2	D_3	Supply	Penalty
II	$\boxed{20}$ 3	$\boxed{20}$ 2	40/20	1
III	$\boxed{20}$ 2	5	20/0	3 ← (max)
Demand	40/20	20/0	60	
Penalty	1	3		

Minimum cost $= 1 \times 10 + 1 \times 20 + 1 \times 10 + 3 \times 20 + 2 \times 20 + 2 \times 20$

$= 10 + 20 + 10 + 60 + 40 + 40$

$= 180$ Rs.

Initial Basic Feasable Solution —

$x_{11} = 20$ $\quad\quad$ $x_{23} = 20$

$x_{13} = 10$ $\quad\quad$ $x_{24} = 10$

$x_{22} = 20$ $\quad\quad$ $x_{32} = 20$

(iii) To test the optimality —

	D_1	D_2	D_3	D_4		
I	$\boxed{20}$ 1	2	$\boxed{10}$ 1	4	30	u_1
II	3	$\boxed{20}$ 3	$\boxed{20}$ 2	$\boxed{10}$ 1	50	u_2
III	4	$\boxed{20}$ 2	5	9	20	u_3
	20	40	30	10	100	
	v_1	v_2	v_3	v_4		

$m+n-1 = 3+4-1 = 6 =$ Occupied cells

Calculate for occupied cells—

$C_{ij} = u_i + v_j$

$C_{11} = u_1 + v_1 \Rightarrow 1 = u_1 + v_1$

$C_{13} = u_1 + v_3 \Rightarrow 1 = u_1 + v_3$

$C_{22} = u_2 + v_2 \Rightarrow 3 = u_2 + v_2$

$C_{23} = u_2 + v_3 \Rightarrow 2 = u_2 + v_3$

$C_{24} = u_2 + v_4 \Rightarrow 1 = u_2 + v_4$

$C_{32} = u_3 + v_2 \Rightarrow 2 = u_3 + v_2$

Put $u_2 = 0$

$v_2 = 3$	$u_1 = -1$
$v_3 = 2$	$u_3 = -1$
$v_4 = 1$	$v_1 = 2$

Calculate for non-occupied cells—

$d_{ij} = C_{ij} - (u_i + v_j)$

$d_{12} = 2 - (u_1 + v_2) \Rightarrow 2-(-1+3) = 0$

$d_{14} = 4 - (u_1 + v_4) \Rightarrow 4-(-1+1) = 4$

$d_{21} = 3 - (u_2 + v_1) \Rightarrow 3-(0+2) = 1$

$d_{31} = 4-(u_3+v_1) \Rightarrow 4-(-1+2) = 3$
$d_{33} = 5-(u_3+v_3) \Rightarrow 5-(-1+2) = 4$
$d_{34} = 9-(u_3+v_4) \Rightarrow 9-(-1+1) = 9$

$d_{ij} \geq 0$

The basic feasable solution is optimal.
Minimum transportation cost = Rs. 180.

Ques 3:- A company has 4 machines to do 3 jobs. Each can be assigned to one and only one machine. The cost of each job on each machine is given in the following table –

Machine

Job	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

Sol:- Since the given problem is an unbalanced assignment problem, so we add a dummy row.

	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22
D	0	0	0	0

Now the problem can be solved by usual method.

Step 1- Subtract the smallest element of each row from every element of the corresponding row in the matrix

	W	X	Y	Z
A	0	6	10	14
B	0	5	9	11
C	0	5	9	12
D	0	0	0	0

Step 2– Make the assignment.

	W	X	Y	Z	
A	$\boxed{0}$	6	10	14	✓
B	$\cancel{0}$	5	9	11	✓
C	$\cancel{0}$	5	9	12	✓
D	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	
	✓				

Step 3–

	W	X	Y	Z	
A	$\boxed{0}$	1	5	9	✓
B	$\cancel{0}$	$\boxed{0}$	4	6	✓
C	$\cancel{0}$	$\cancel{0}$	4	7	✓
D	5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	
	✓	✓			

Step 4–

	W	X	Y	Z
A	$\boxed{0}$	1	1	5
B	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2
C	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	3
D	9	4	$\cancel{0}$	$\boxed{0}$

$A \rightarrow W, \; B \rightarrow X, \; C \rightarrow Y, \; D \rightarrow Z$

Total minimum cost = ~~18 + 13 + 19 + 0~~

~~= 50 Rs.~~

Ques4:- Find the optimal solution for the assignment problem–

	I	II	III	IV	V
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

Sol:- Step 1- Subtract the smallest element of each row from every element of the corresponding row in the given matrix.

	I	II	III	IV	V
A	3	9	0	8	12
B	3	1	6	0	9
C	1	4	3	0	4
D	4	7	0	11	9
E	4	0	2	1	5

Step 2- Subtract the smallest element of each column from every element of the corresponding column in the given matrix.

	I	II	III	IV	V
A	2	9	0	8	8
B	2	1	6	0	5
C	0	4	3	0	0
D	3	7	0	11	5
E	3	0	2	1	1

Step 3- Make the assignment.

	I	II	III	IV	V	
A	2	9	$\boxed{0}$	8	8	✓
B	2	1	6	$\boxed{0}$	5	
C	$\boxed{0}$	4	3	$\cancel{0}$	$\cancel{0}$	
D	3	7	$\cancel{0}$	11	5	✓
E	3	$\boxed{0}$	2	1	1	
			✓			

Step 4 –

	I	II	III	IV	V
A	$\boxed{0}$	7	$\cancel{0}$	6	6
B	2	1	8	$\boxed{0}$	5
C	$\cancel{0}$	4	5	$\cancel{0}$	$\boxed{0}$
D	1	5	$\boxed{0}$	9	3
E	3	$\boxed{0}$	4	1	1

$A \to I$, $B \to IV$, $C \to V$, $D \to III$, $E \to II$

Total minimum cost $= 11 + 6 + 16 + 17 + 10$

$= 60$ Rs.

Ans.

3/4/19

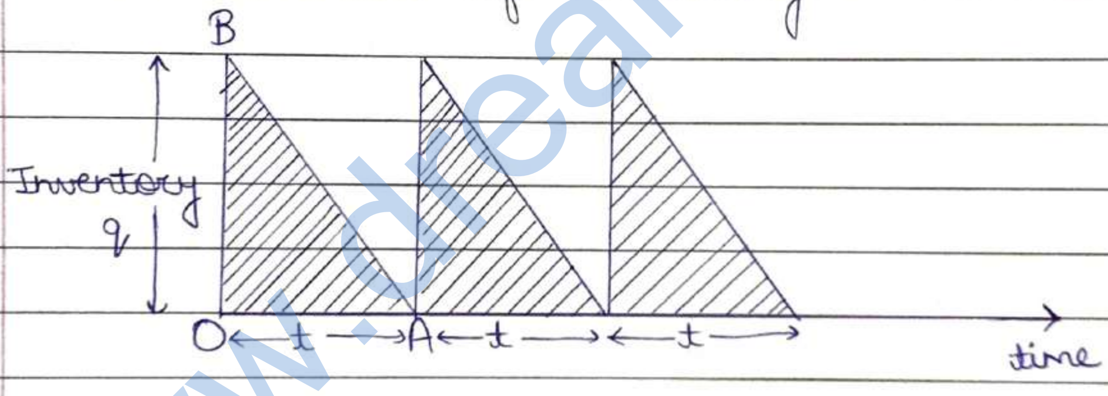
* Model I :-

To drive economic lot size formula and the minimum average cost under the following assumptions –

1- Demand is uniform at a rate.
2- Production is instantaneous (i.e. production rate ~~of~~ is infinite.
3- Lead time is 0.
4- C_1 = holding cost per unit of quantity per unit of time.
5- C_3 = set-up cost per production run.
6- Shortage are not allow.

Proof – Let q be the units of quantity produced per production run at interval of time t.

The situation of inventory –



The demand rate is r units per unit time. The total demand is 1. Run of time interval t is rt.

The quantity $q = rt$ — (1) (Shortage are not allowed)

The cost of holding inventory = C_1 (Area of OAB)

$$= C_1 \times \frac{1}{2} q t$$

Set-up cost = C_3

The total cost per production run of time t

$$= \frac{1}{2} C_1 q t + C_3$$

The average total cost per unit time $C(q)$

$$= \frac{1}{2} C_1 q + \frac{C_3}{t}$$

From (1), $t = \frac{q}{r}$

$$C(q) = \frac{1}{2} C_1 q + \frac{C_3 r}{q} \quad \text{— (2)}$$

This equation is known as cost equation.

For minimum value -

$$\frac{dC}{dq} = \frac{1}{2} C_1 - \frac{C_3 r}{q^2}$$

Put $\frac{dC}{dq} = 0$

$$\frac{1}{2} C_1 - \frac{C_3 r}{q^2} = 0$$

$$\frac{1}{2} C_1 = \frac{C_3 r}{q^2}$$

$$q^2 = \frac{2 C_3 r}{C_1}$$

$$\boxed{q = \sqrt{\frac{2 C_3 r}{C_1}}}$$

Again diff. $\frac{d^2 C}{dq^2} = \frac{2 C_3 r}{q^3}$ (+ve)

For minimum $\boxed{q = q^* = \sqrt{\frac{2 C_3 r}{C_1}}}$

This is the economic lot size formula.

Put in eq (1)

$$t = \frac{q}{r}$$

$$t = \frac{1}{r} \sqrt{\frac{2 C_3 r}{C_1}}$$

$$t = \frac{1}{r} \times \sqrt{r} \sqrt{\frac{2 C_3}{C_1}}$$

$$t = \sqrt{\frac{2C_3}{C_1 r}}$$

$$\boxed{t = t^* = \sqrt{\frac{2C_3}{C_1 r}}}$$

The minimum cost per unit time is given by-
From eq(2)

$$C_{min} = \frac{1}{2}C_1 q + \frac{C_3 r}{q}$$

$$C_{min} = \frac{1}{2}C_1\sqrt{\frac{2C_3 r}{C_1}} + C_3 r\sqrt{\frac{C_1}{2C_3 r}}$$

$$C_{min} = \frac{1}{2}C_1\frac{\sqrt{2C_3 r}}{\sqrt{C_1}} + C_3 r\frac{\sqrt{C_1}}{\sqrt{2}\times\sqrt{C_3 r}}$$

$$C_{min} = \frac{1}{2}\sqrt{C_1}\sqrt{2C_3 r} + \sqrt{C_3 r}\times\frac{\sqrt{C_1}}{\sqrt{2}}$$

$$C_{min} = \frac{1}{\sqrt{2}}\sqrt{C_1 C_3 r} + \frac{1}{\sqrt{2}}\sqrt{C_1 C_3 r}$$

$$C_{min} = \frac{2}{\sqrt{2}}\sqrt{C_1 C_3 r}$$

$$\boxed{C_{min} = \sqrt{2C_1 C_3 r}}$$

If C_1 and C_3 are constant then the minimum cost per unit time is propotional to the square root of the demand rate.

Ques- A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the shortage cost amount to Rs. 0.60 per unit per year. The set-up cost per run is Rs. 80.00. Find the optimum run size and the minimum average yearly cost.

Sol:- $C_1 = 0.60$ Rs.

$C_3 = 80.00$ Rs.

$r = 600$

$$q = \sqrt{\frac{2C_3 r}{C_1}} = \sqrt{\frac{2 \times 80 \times 600 \times 100}{0.60}} = \sqrt{160000}$$

$$\boxed{q = 400}$$

$$t = \frac{q}{r} = \frac{400}{600} = \frac{2}{3} \times 12$$

$t = 8$ months

$$C_{min} = \sqrt{2C_1C_3 r}$$

$$= \sqrt{2 \times \frac{0.60}{100} \times 80 \times 600} = \sqrt{57600}$$

$C_{min} = 240$ unit / year

Ques- The shortage cost of one item is Rs. 1.00 per month and the setup cost is Rs. 25.00 per run. If the production is instantaneous and the demand is 200 unit per month. Find the optimal size of the batch and best time for the replacement of inventory.

Sol:
$C_1 = $ Rs. 1
$C_3 = $ Rs. 25
$r = 200$

$$q = \sqrt{\frac{2C_3 r}{C_1}} = \sqrt{\frac{2 \times 25 \times 200}{1}} = \sqrt{10000}$$

$$\boxed{q = 100}$$

$$t = \frac{q}{r} = \frac{100}{200} = \frac{1}{2} \text{ month}$$

$$C_{min} = \sqrt{2C_1C_3 r}$$

$$= \sqrt{2 \times 1 \times 25 \times 200} = 100 \text{ unit / month}$$

$$C_{min} = 100 \times 12$$

$$= 1200 \text{ unit / year.}$$

* Another form of model I :- To drive an economic lot size formula for the following assumptions –

(i) λ = the demand for product in one unit of time (say 1 year).

(ii) Production rate is infinite.

(iii) Lead time is zero.

(iv) P = Price of one time of product in rupees.

(v) I = Cost of carrying one rupee to the inventory for one year.

(vi) C_3 = Set-up cost per order (per cycle).

(vii) Shortages not allowed.

Sol:- Let q be the units of quantity production per cycle (in time t).

From model I, total inventory in one cycle

$$= \frac{1}{2} q t.$$

The average inventory $\frac{q}{2}$. It will continue through out the year holding cost per item for 1 year $= IP$

The total holding cost per year $= \frac{1}{2} q IP$

Setup cost in one year $= \frac{\lambda}{q} C_3$

The total cost in one year $C(q) = \frac{1}{2} q IP + \frac{\lambda}{q} C_3$

For minimum $\frac{dC}{dq} = \frac{1}{2} IP + \frac{\lambda}{q^2} C_3$

Now put $\frac{dC}{dq} = 0$

$$\frac{1}{2} IP - \frac{\lambda}{q^2} C_3 = 0$$

$$\frac{\lambda}{q^2} C_3 = \frac{1}{2} IP$$

$$q = \sqrt{\frac{2\lambda C_3}{IP}}$$

again diff. $\frac{d^2c}{dq^2} = \frac{2\lambda}{q^3} C_3$ which is positive

* Model II :-

$$q = \sqrt{\frac{2RC_3}{C_1 t}}$$

R is total demand in the total period.
It is the economic lot size formula.
Minimum cost per unit time :

$$C_{min} = \frac{1}{2} C_1 \sqrt{\frac{2RC_3}{C_1 t}} + \frac{RC_3}{t} \sqrt{\frac{C_1 t}{2RC_3}}$$

$$C_{min} = \sqrt{\frac{2 C_1 C_3 R}{t}}$$

* Model III :-

Economic lot size formula

$$q = q^* = \sqrt{\frac{2 C_3 r k}{C_1 (k-r)}}$$

Minimum cost per unit time is given by

$$C_{min} = \sqrt{2 C_1 C_3 r \left(\frac{k-r}{r}\right)}$$

Also time of one run is given by $t = \sqrt{\frac{2 C_3 k}{C_1 r (k-r)}}$

Ques– A manufacturer has to supply 12000 units of a product per year to each customer. The demand is fixed and known. Shortage cost is assumed to be infinite. The inventory holding cost is Rs 0.20 per unit / month. and the setup cost per run is Rs. 350, then find–

(i) The optimum run size q_0

(ii) Optimum scheduling period t_0

(iii) Minimum total variable yearly cost.

Sol:–

$C_1 = 0.20$ Rs. $\quad r = \frac{\overset{1000}{\cancel{12000}}}{\cancel{12}} = 1000$ / month

$C_3 = 350$ Rs.

(i) $q_0 = \sqrt{\frac{2C_3 r}{C_1}} = \sqrt{\frac{2 \times 350 \times 1000 \times 10\cancel{0}}{0.2\cancel{0}}}$

$= \sqrt{3500000} = 1870.8$ units / run

(ii) $t_0 = \frac{q}{r}$

$t_0 = \frac{1870.8}{1000} = 1.870$

(iii) $C_{min} = \sqrt{2C_1C_3 r}$

$= \sqrt{2 \times \frac{0.2\cancel{0}}{10\cancel{0}} \times 35\cancel{0} \times (1000 \cancel{\times 12})}$

$= \sqrt{140000}$

$= 374.16 \times 12$

$C_{min} =$ Rs. 4490 / year